PS Complexity of Polynomial-Time Problems

https://www.cosy.sbg.ac.at/~sk/courses/polycomp/

Exercise sheet 4 Due: Sunday, December 18, 2017

Total points : 40

Prove all your claims!

Exercise 1 (*10 points*) Let A and B be two $n \times n$ matrices with integer entries in $\{-M, -M+1, \dots, M-1, M\}$. Show that the min-plus matrix product of A and B can be computed in time $O(M^2 \cdot n^{\omega})$. Here ω is the exponent of matrix multiplication.

Remark: This problem can actually be solved in time $(M \cdot n^{\omega} \cdot \text{poly}(\log M, \log n))$.

Exercise 2 (*10 points*) In the k-**Clique** problem, we are given an unweighted undirected graph G and are asked to decide if G contains a k-clique (i.e., a set of k vertices which are pairwise adjacent). Show that if k is divisible by 3, then k-**Clique** can be solved in time $O(n^{\frac{\omega k}{3}})$.

Hint: Reduce the problem to detecting a triangle in a graph with $O(n^{\frac{k}{3}})$ vertices.

Remark: This is the best running time known for this problem.

Exercise 3 (*10 points*)

In the **Zero-Weight 3-Star** problem, we are given a weighted 4-partite graph $G = (V_1 \cup V_2 \cup V_3 \cup V_4, E)$, where $|V_1| = |V_2| = |V_3| = |V_4| = n$, and are asked to decide whether there are $v_1 \in V_1$, $v_2 \in V_2$, $v_3 \in V_3$, and $v_4 \in V_4$ such that $w(v_1, v_2) + w(v_1, v_3) + w(v_1, v_4) = 0$.

Show that **Zero-Weight 3-Star** has an algorithm with running time $O(n^{3-\epsilon})$ (for some $\epsilon > 0$) if and only if there is an algorithm with running time $O(n^{3-\delta})$ (for some $\delta > 0$) that decides if at least one of n given **3SUM** instances has a solution.

Exercise 4 (*10 points*)

In the **Segment Visibility** problem, we are given a set S of n line segments in the plane and two distinguished line segments a and b, and are asked to decide whether there are points p on a and q on b such that the line through the points p and q does not intersect any line segment in S (i.e., we want to check whether a is "visible" from b).

Show that if **Segment Visibility** can be solved in them $O(n^{2-\epsilon})$ for some $\epsilon > 0$, then **3SUM** can be solved in time $O(n^{2-\delta})$ for some $\delta > 0$.