Orthogonal Vectors Problem

Def OVP
Given: Set $A, B \in \{0, 1\}^d$, $n = |A| = |B|
Tools: Decide if there exists $a \in A$, $b \in B$ s.t. $a \perp b$

Algorithms: $O(n^3)$ trivial
  - Fastest known $O(n^{2.376})$ if $d = \log n$

OPEN Is there an $O(n^{2+\epsilon})$-time alg for some constant $\epsilon > 0$?

Theorem: If there was an $O(n^{2+\epsilon})$-time alg for diameter,
then there would be an $O(n^{2+\epsilon})$-time alg for OVP.

Proof: Let $G_{AB}$ be the graph

Def Diameter Problem
Given (Unweighted) Graph $G = (V, E)$, $n = |V|$, $m = |E|
Task: Compute diameter of $G$, $\text{diam}(G) = \max_{u,v \in V} \text{dist}_G(u, v)$

Simple alg: - BFS for every node $u$. (BFS gives $\text{dist}(u, \cdot)$ only)
  - Determine max over all pairs $O(n^2)$
OPEN $O(n^{m^{1/3}})$-alg for diameter?

$	ext{diam}$
Proof c't'd

If there is 0'th pair, say \( a \in A, b \in B \)

Whenever \( a[k]=1 \), we have \( b[k]=0 \)

We check that no path from \( a \) to \( b \) has length < 3

Enumerate all possibilities (simple paths)
- \( a-l-a'-k \) (for some \( a \in A \) and some \( k \))
- \( a-l-k-r \) (for some \( k \))
- \( a-k-b' \) ... then \( b' \neq b \)

None of these paths of length < 3 above includes \( b \)

\[ \Rightarrow \text{dist}(a,b) \geq 3 \] with OSS

\[ \text{diam}(G) \geq 3 \Rightarrow \text{diam}(G) = 3 \]

\[ \times = O(n^2 \cdot \text{polylog}(d)) \]