PS Geometric Modeling: Homework Assignment Sheet 1 for 2024-03-22
Assignment 1.1 Consider $f, g: \mathbb{N} \rightarrow \mathbb{R}$ with $f(n):=\log n+2 n^{2}+2 n+4$ and $g(n):=n^{2}$. Which of the following claims are correct?
a) $f \in O(g)$,
b) $f \in \Theta(g)$,
c) $f \in \Omega(g)$,
d) $f \in o(g)$

Assignment 1.2 Use induction to prove that $3^{n}>n \cdot 2^{n}$ for all $n \in \mathbb{N}_{0}$.
Assignment 1.3 Use case analysis to solve the equation $|x+2|+|x+5|=7$ over $\mathbb{R}$.
Assignment 1.4 Prove or disprove: If two circles have three or more distinct points in common then the circles are identical.

Assignment 1.5 For $n \in \mathbb{N}$, let $y_{n}:=\frac{\log n+7 \sqrt{n}-10}{n^{2}}$. Prove that $\lim _{n \rightarrow \infty} y_{n}=0$.

## PS Geometric Modeling: Homework Assignment Sheet 2 for 2024-04-12

Assignment 2.1 Consider $k$ real numbers $a_{1}, a_{2}, \ldots, a_{k} \in \mathbb{R}$, together with some $m, n \in \mathbb{N}$ such that $1 \leqslant m, n \leqslant k$. The following definition

$$
\sum_{i=m}^{n} a_{i}:=\left\{\begin{array}{cl}
0 & \text { if } n<m \\
a_{m} & \text { if } n=m \\
\left(\sum_{i=m}^{n-1} a_{i}\right)+a_{n} & \text { if } n>m
\end{array}\right.
$$

belongs to one of two main classes of definitions. What are the characteristics of this class of definitions?
Assignment 2.2 Let $G$ be a set with three elements $n, a, b$. (That is, $G=\{n, a, b\}$.) Let $\star$ be a binary operation on $G$. Prove or disprove: If $(G, \star)$ is a group with neutral element $n$ then its multiplication table is uniquely determined and is given below:

| $\star$ | $n$ | $a$ | $b$ |
| :---: | :---: | :---: | :---: |
| $n$ | $n$ | $a$ | $b$ |
| $a$ | $a$ | $b$ | $n$ |
| $b$ | $b$ | $n$ | $a$ |

Assignment 2.3 Prove that for all $n, k, i \in \mathbb{N}_{0}$, with $i \leqslant k \leqslant n$,

$$
\frac{\binom{n-i}{k-i}}{\binom{n}{k}}=\frac{\binom{k}{i}}{\binom{n}{i}} .
$$

Assignment 2.4 Consider vectors $\nu_{1}, \nu_{2}, \ldots, \nu_{k}$ of a vector space $V$ over $\mathbb{R}$. Prove explicitly in a clean formal way that

$$
\sum_{i=1}^{k} \lambda_{i} \nu_{i}=0 \quad \Rightarrow \quad \lambda_{1}=\lambda_{2}=\cdots=\lambda_{k}=0
$$

for all $\lambda_{1}, \ldots, \lambda_{k} \in \mathbb{R}$ if $\nu_{1}, \nu_{2}, \ldots, \nu_{k}$ are linearly independent.
Assignment 2.5 Consider a vector space $V$ over $\mathbb{R}$ and two finite sets $B, B^{\prime}$ with $B \subseteq V$ and $B^{\prime} \subseteq V$. Prove explicitly: If $B \subset B^{\prime}$ then at least one of $B, B^{\prime}$ does not form a basis of $V$.

