PS Computational Geometry: Assignment Sheet 1 for 11-Oct-2024

Assignment 1.1 Consider $f, g : \mathbb{N} \to \mathbb{R}^+$ and suppose that $\lim_{n\to\infty} f(n)/g(n)$ exists. Prove that

$$f \in o(g) \quad \Longleftrightarrow \quad \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0.$$

Assignment 1.2 Consider $f, g : \mathbb{N} \to \mathbb{R}^+$. Which of the following claims hold?

(1) $f \in \Theta(g) \Longrightarrow (\exists c \in \mathbb{R}^+ \ \lim_{n\to\infty} \frac{f(n)}{g(n)} = c)$

(2) $(\exists c \in \mathbb{R}^+ \ \lim_{n\to\infty} \frac{f(n)}{g(n)} = c) \Longrightarrow f \in \Theta(g)$

Assignment 1.3 Prove – or, at least, argue convincingly – that Euler's formula holds for all polyhedra of genus zero.

Assignment 1.4 Consider a planar graph, count the number of its vertices that are of odd degree, and let k be that number. Prove or disprove: The number k is always even.