PS AADS: Homework Assignment Sheet 1 for 2025-Oct-09

Assignment 1.1 Solve the following equations (in x) over $\mathbb{R}$ without using a calculator or computer algebra system (like Mathematica).

(a) $\log x = 3$

(b) $\log_x 4096 = 4$

(c) $x \sum_{i=0}^{\infty} \frac{1}{9^i} = 9$

(d) $e^{2\ln 3 - 3\ln 2} = \frac{9}{x}$

(e) $\log(x-1) + \log(x+1) - 2\log(x) = \log \frac{63}{64}$

(f) $x^4 - x^3 - 4x^2 + 4x = 0$

Assignment 1.2 Which set-theoretic relations hold among the growth rates $2^{O(n)}$, $O(2^{n^k})$ for some $k \in \mathbb{N}$, and $O(2^n)$? Do we get equality?

Assignment 1.3 Let $f, g : \mathbb{N} \to \mathbb{R}^+$. Prove or disprove the following claim (relative to the definition of $o(\cdot)$ given in Def. 29):

$$g \in o(f) \quad \Leftrightarrow \quad \lim_{n\to\infty} \frac{g(n)}{f(n)} = 0.$$

Assignment 1.4 Let $f, g\colon \mathbb{N} \to \mathbb{R}^+$ grow monotonously and unboundedly. Prove or disprove:

(1) If $g \in o(f)$ then $\log g \in o(\log f)$.

(2) If $\log g \in o(\log f)$ then $g \in o(f)$.

Assignment 1.5 Prove that

(1) $2^n \in o(n!)$, and

(2) $n! \in o(2^{n^{1+c}})$ for all $c > 0$.

Assignment 1.6 Determine $g\colon \mathbb{N} \to \mathbb{R}$ such that $T \in \Theta(g)$ if

(a) $T(n) = 27 \cdot T\left(\frac{n}{9}\right) + 7n\sqrt{n}$,

(b) $T(n) = 9 \cdot T\left(\frac{n}{3}\right) + 7n^{\sqrt{3}}$.

Assignment 1.7 Determine $g\colon \mathbb{N} \to \mathbb{R}$ such that $T \in \Theta(g)$ if $T(n) = 27 \cdot T\left(\frac{n}{9}\right) + \sum_{i=1}^{n} i$.