Brief Announcement: A Note on Hardness of Diameter Approximation

Karl Bringmann[1] Sebastian Krinninger[2]

[1]Max Planck Institute for Informatics
Saarland Informatics Campus

[2]Department of Computer Sciences
University of Salzburg

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Motivation

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  Upper: 3/2-approximation in $O(\sqrt{n \log n} + D)$ rounds [Holzer et al. '14]
  Lower: $(3/2 - \varepsilon)$-approximation in $\tilde{\Omega}(n)$ rounds [Abboud et al. '16]

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  Lower: $(3/2 - \varepsilon)$-approximation in $\tilde{\Omega}(n)$ rounds [Abboud et al. '16]

Goal: Fine-grained understanding of hardness of diameter approximation

Several recent works in CONGEST model and RAM model:
[Frischknecht at al. '12, Roditty/Williams '13, Chechik et al. '14, Holzer et al. '14, Cairo et al. '16, Abboud et al. '16]

Our Results: CONGEST Model

Theorem

In the CONGEST model, any algorithm distinguishing between graphs of diameter $2\ell + q$ and graphs of diameter $3\ell + q$ when $\ell \geq 1$ and $q \geq 1$ requires $\tilde{\Omega}(n)$ rounds.

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Theorem ([Abboud et al. '16])

In the CONGEST model, any algorithm distinguishing between graphs of diameter $4\ell + 1 + q$ and graphs of diameter $6\ell + q$ when $\ell \geq 1$ and $q \geq 1$ requires $\tilde{\Omega}(n)$ rounds.

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2 vs. 3 is hard [Frischknecht et al. '12]

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In the RAM model, under the Orthogonal Vectors Hypothesis, there is no algorithm distinguishing between graphs of diameter $2\ell + q$ and graphs of diameter $3\ell + q$, where $\ell \geq 1$ and $q \geq 0$, in time $O(m^{2-\delta})$ for any constant $\delta > 0$.

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Strong Exponential Time Hypothesis $\Rightarrow$ Orthogonal Vectors Hypothesis

Our Approach

Orthogonal Vectors Problem:

Given sets $A, B \subseteq \{0, 1\}^d$, decide if there are $a \in A$ and $b \in B$ such that $a \perp b$

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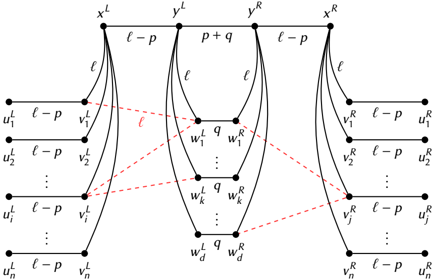
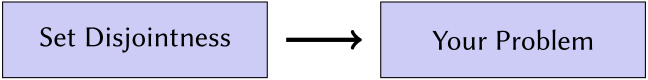
CONGEST model: Reduce OV to Diameter



Take-Home Message

CONGEST model lower bounds:



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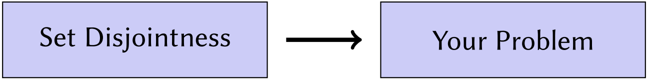

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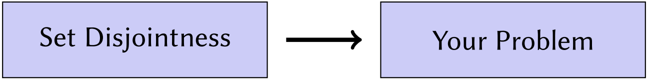

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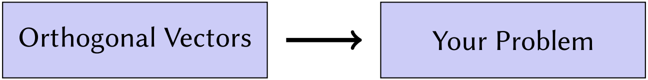

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Set Disjointness $\longrightarrow$ Your Problem

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Suggestion:

Set Disjointness $\longrightarrow$ Orthogonal Vectors $\longrightarrow$ Your Problem