Today
- Orga
- k-SAT
- Hitting Set
- ~~OV~~

Sat of Boolean Formulas

$\wedge \ \vee \ \neg \ \not\to$

Def. (SAT)

Input: Bool. Formula φ

Task: Decide if there is assignment to variables of φ s.t. φ evalates to true

N: # variables

CNF-SAT: $\varphi = C_1 \wedge C_2 \wedge \ldots \wedge C_M$

$C_i = \ell_{i1} \vee \ell_{i2} \vee \ldots \vee \ell_{ik_i}$

$\ell_{ij} = x$ or $\ell_{ij} = \bar{x}$ for some var. x

k-SAT: $C_i = \ell_{i1} \vee \ldots \ell_{ik}$

$(a \vee b \vee \bar{c}) \wedge (\bar{a} \vee \bar{b} \vee d) \wedge (\bar{a} \vee c \vee d)$

Observation: SETH $\Rightarrow$ ETH $\Rightarrow$ $P \neq NP$

Fastest known alg. (Baseline $2^N \cdot poly(M,N)$)

2-SAT: $O(M+N)$

3-SAT: $1.31^N \cdot poly(M,N)$

4-SAT: $1.47^N \cdot poly(M,N)$

[Paturi et al '98]

param. k

$f(k) \cdot poly(n)$

k^k

k-SAT $2^{N(1-c/k)} \cdot poly(M,N)$ for some constant c

CNF-SAT: $2^{(1-\Theta(1/\log(M/N)))N} \cdot poly(M,N)$ [Calabro et al '06]

Conjecture [Impagliazzo/Paturi]: [Exponential Time Hypothesis

$\forall k \geq 3$, $\exists$ constant c_k s.t. no $2^{c_k \cdot N} \cdot poly(M,N)$-time algorithm for k-SAT

($\Rightarrow$ No $2^{o(N)}$-alg. for 3-SAT) (No $2^{o(\sqrt{n})}$-time)

Conjecture [I/P]: Strong Exp Time Hyp. (SETH)

For $\delta > 0$, $\exists k$ s.t. no $2^{(1-\delta)N} \cdot poly$-time for k-SAT

($\Rightarrow$ no $2^{(1-\delta)N} poly(M,N)$-time algorithm for CNF-SAT (for any $\delta > 0$))

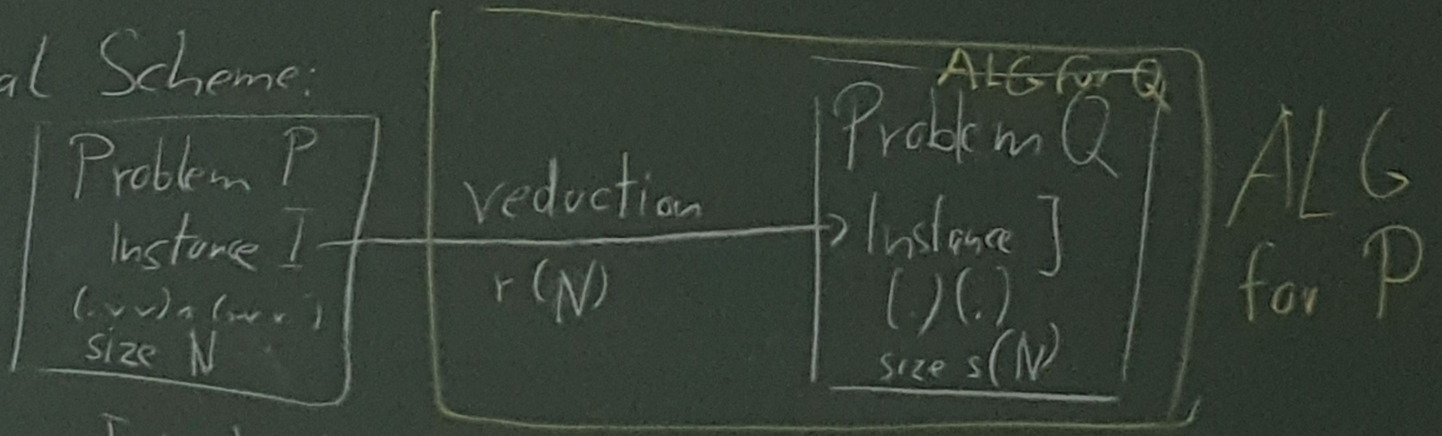
From CNF-SAT to Orthogonal Vectors

OV: Given $A, B \subseteq \{0,1\}^d$, Q: $\exists\, a \in A, b \in B$ s.t. $a \perp b$

<u>Def</u>: Orthogonal Vectors Hypothesis (OVH):
There is alg with running time $n^{2-\varepsilon}\,\mathrm{poly}(d)$ for OV for any $\varepsilon > 0$

<u>Theorem</u>: SETH $\Rightarrow$ OVH

General Scheme:



I is 'yes'-instance $\Longleftrightarrow$ J is a 'yes'-instance

Then $t(n)$-time alg for Q implies $r(N) + t(s(N))$-time alg for P

Or in other words If P has no $r(N) + t(s(N))$-time alg, then Q has no $t(n)$-time alg.

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$\varphi \longrightarrow A, B$ $\qquad \varphi = C_1 \wedge \ldots \wedge C_M$

Varidbles of φ: $x_1, x_2, \ldots x_N$

$\mathcal{U} = \{0,1\}^{N/2}$ all possible truth value assigments to variables $x_1, \ldots, x_{N/2}$

$V = \{0,1\}^{N/2}$ — " — $x_{N/2+1}, \ldots, x_N$

Partial assigment $\underset{(v \in V)}{\overset{u \in \mathcal{U}}{\vee}}$ satisfies clause C if

- $\exists i$ s.t. x_i evaluates to 1 under ass $\overset{v}{u}$ and x_i appears in C
- — " — 0 — " — $\bar{x}_i$ — " —

write: $\mathrm{sat}(\overset{v}{u}, C) = \begin{cases} 1 & \text{if } \overset{v}{u} \text{ sat } C \\ 0 & \text{o.w.} \end{cases}$

$\mathrm{unsa}(\overset{v}{u}, C) = 1 - \mathrm{sat}(\overset{v}{u}, C)$

$A := \{(\mathrm{unsat}(u, C_1), \ldots, \mathrm{unsat}(u, C_M)) \mid u \in \mathcal{U}\}$

$B := \{(\mathrm{unst}(v, C_1), \ldots, \mathrm{unsat}(v, C_M)) \mid v \in V\}$

Claim: φ sat. $\iff$ A, B has orth pair

Proof $\Rightarrow$ Consider satisfying assignment w^* of φ

Let u^* and v^* be partial assigments on $\underset{\text{first } N/2 \text{ var}}{\downarrow}$ $\underset{\text{second } N/2 \text{ var}}{\searrow}$ s.t. $w^* = (u^*, v^*)$

Then for every clause C:

u^* sat. C or v^* sat C

$\iff \mathrm{sat}(u^*, C) = 1$ or $\mathrm{sat}(v^*, C) = 1$

$\iff \mathrm{unsat}(u^*, C) = 0$ or $\mathrm{unsat}(v^*, C) = 0$

$\iff \exists$ vectors in A, B (corr to u^* and v^*) that are orthogonal

Time Complexity: size OV instance

$|V| = |\mathcal{U}| = 2^{N/2} = |A| = |B| \qquad d = M$

Running for creating OV instance $2^{N/2} \cdot M$

Conjecture [I/P] Strong Exp Time Hyp (SETH)

For $\delta > 0$, $\exists k$ s.t. no $2^{(1-\delta) \cdot N} \cdot \mathrm{poly}$-time for k-SAT

($\Rightarrow$ no $2^{(1-\delta) N} \mathrm{poly}(M, N)$-time algorithm for CNF-SAT (for any $\delta > 0$))

Theorem Assuming SETH, there is no $n^{2-\varepsilon}$ poly(d)-time alg. for OV [Or SETH $\Rightarrow$ OVH]

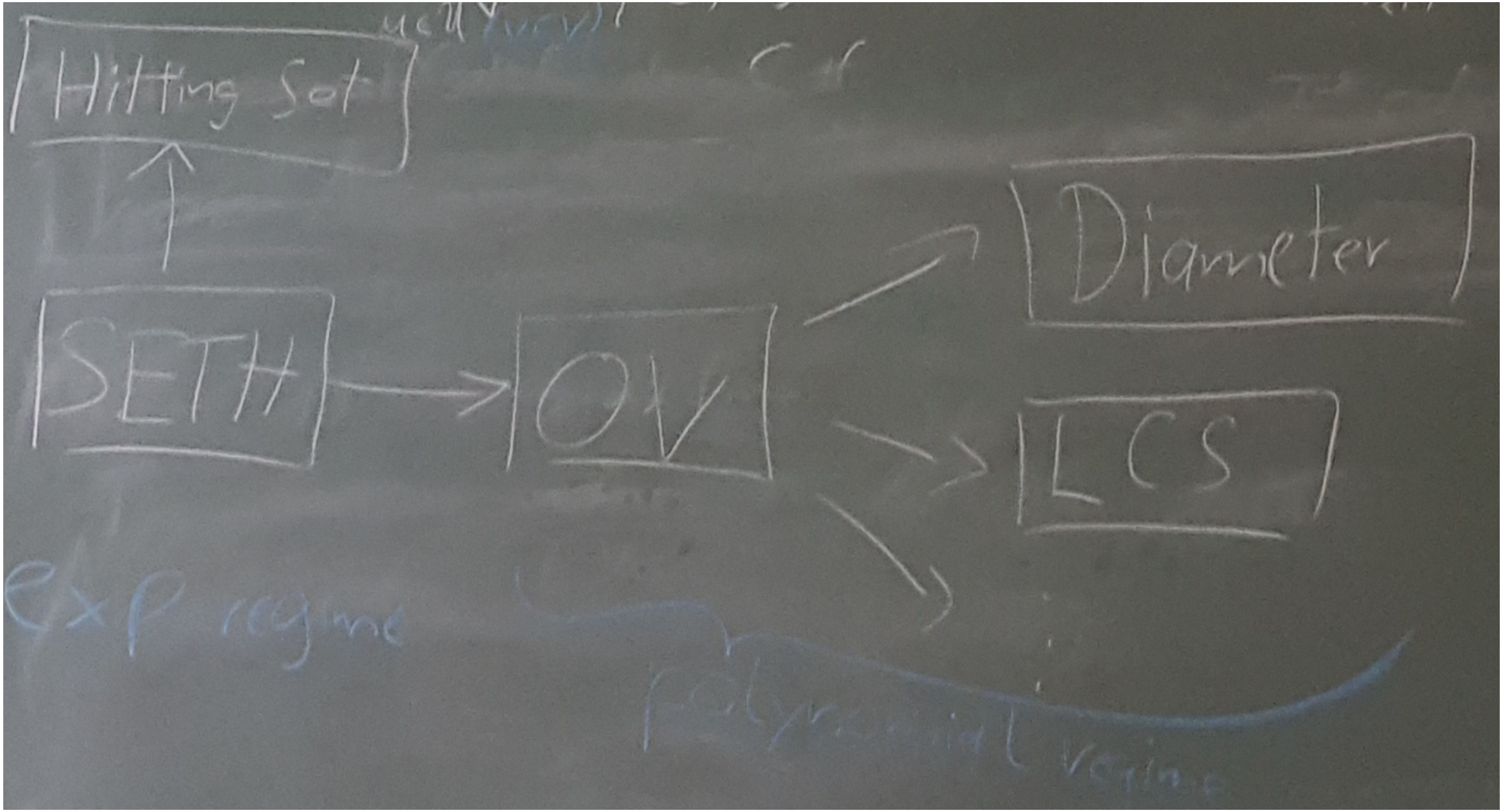
Proof: $n^{2-\varepsilon}$ poly(d) time

Then for our instances $n = 2^{N/2} \cdot M$ $\quad d = M$

$$n^{2-\varepsilon}\,\mathrm{poly}(d) = \left(2^{N/2} M\right)^{2-\varepsilon} \cdot \mathrm{poly}(M)$$

$$= 2^{(2-\varepsilon)\frac{N}{2}}\,\mathrm{poly}(M)$$

$$= 2^{(1-2\varepsilon)N} \cdot \mathrm{poly}(M)$$ violates SETH


Hitting Set
SETH
OV
Diameter
LCS
exp regime
polynomial regime