Zerotree Image Compression using Anisotropic Wavelet Packet Transform

Rade Kutil

University of Salzburg, Dept. of Scientific Computing, Austria

ABSTRACT

The anisotropic wavelet packet transform is an extension of the conventional wavelet (packet) transform where the basis can have different scales in different dimensions. As there are certain kinds of images with different behaviour in horizontal and vertical direction, anisotropic wavelet packet bases can be adapted more precisely to these images. Zero-tree image compression has already proved its efficiency on conventional wavelet transformed data as well as for wavelet packets. In this work, zero-tree methods are extended to work with anisotropic wavelet packets and coding results are shown for several types of images.

1. INTRODUCTION

Image and video coding methods that use wavelet transforms have been successful in providing high rates of compression while maintaining good image quality and have generated much interest in the scientific community as competitors to DCT based compression schemes in the context of the MPEG-4[1] and JPEG2000[2] standardisation processes. SPIHT[3] and SMAWZ[4] are wavelet based image compression algorithms that use so-called zero-trees[5] (sets of insignificant, small coefficients). Both are simple, fast and competitive in terms of rate-distortion performance and have been extended to work with wavelet packets.

Conventional multidimensional wavelets and wavelet packets have the same scale in each direction. These conventional wavelet packets can therefore be called isotropic wavelet packets (or wavelet packets with isotropic dilations[6]). Several coding algorithms have been developed which use wavelet packets.[4, 7–10] If we consider bases of wavelet packets whose scales are different in different directions, then the number of sub-bands and the number of bases increases substantially.

Additionally, the method of representing specific bases by a decomposition tree (including all intermediate sub-bands) is not applicable any more. To avoid ambiguous representations of equal bases and redundant calculations, a graph structure called *bush*[11, 12] has been introduced as a generalisation of trees. As many algorithms used in wavelet packet image coding (such as the best-basis algorithm or the coding scheme for the decomposition structure) rely on a tree structure, these algorithms have to be modified.

To apply the methods of zero-trees to wavelet packets, so called *similarity trees*[4] have been developed. Recursive rules are used to construct these trees which connect sub-bands (i.e. leaves of the decomposition tree) in a way so that similar sub-bands are close to each other – thus enabling the applicability of zero-trees. Because bushes substitute decomposition trees in the case of anisotropic wavelet packets, new rules to construct similarity trees have to be found.

It is known that certain types of images and textures can be represented very efficiently by wavelet packets. I.e., good wavelet packet bases lead to high information concentration of transformed data. Typically, such images contain regularly repeating patterns. Whenever these patterns behave differently in horizontal and vertical direction, anisotropic transforms can be expected to perform better than isotropic transforms.

This paper first explains the difficulties involved in the efficient generation, representation and storage of anisotropic wavelet packet bases and develops solutions to these problems such as the generalisation of the decomposition tree structure and a best-basis algorithm as well as a redundancy free coding algorithm for the decomposition structure, both relying on the generalised tree structure.

Second, a brief introduction into the SMAWZ zero-tree coding algorithm will be given. It will be shown how this algorithm can be adapted to anisotropic wavelet packets by a modification of the similarity trees of sub-bands that determine the way zero-trees are built. Finally, compression results for a few representative images are shown and compared to the pyramidal and the isotropic wavelet packet decomposition.

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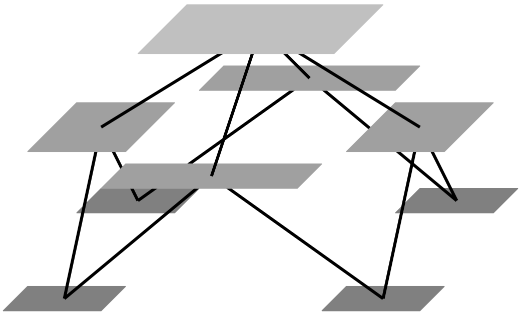


Figure 1. A full bush of anisotropic wavelet packets with decomposition depth 1 in each direction

2. ANISOTROPIC WAVELET PACKET TRANSFORM

Wavelet transforms can efficiently be implemented by the application of a pair of quadrature mirror filters. The original data is divided into a low-pass and a high-pass sub-band each of which is down-scaled by 2. In the 2-D case this procedure is applied in horizontal and vertical directions, producing 4 sub-band. If only the double low-pass filtered sub-band is decomposed further recursively, the resulting decomposition is called pyramidal. If all sub-bands can be decomposed, it is called wavelet packet decomposition. The condition that the sub-bands are always either decomposed in both directions or not decomposed at all guarantees that the base functions (wavelets) have the same scale in each direction (isotropic wavelet packets). If this condition is dropped, sub-bands with anisotropic wavelet packets can be produced which are filtered in only one direction.

The fact that in the anisotropic case the method of representing specific bases by a decomposition tree (including all intermediate sub-bands) is not applicable any more can be explained as follows. Because filtering steps can be interchanged almost arbitrarily, more than one decomposition path can be found for most sub-bands. For instance: $L_h H_v$ and $H_v L_h$, where L_h denotes horizontal low-pass filtering and H_v vertical high-pass filtering, are two paths that produce the same sub-band.

To avoid ambiguous representations of equal bases and redundant calculations, we introduce a graph structure called *bush* that is a generalisation of trees. Figure Figure 1 shows an example of a bush of anisotropic wavelet packets. The basic idea of bushes is to include all possible decomposition paths for each leaf sub-bands (non-decomposed sub-band) in the bush structure. In this way, a bush serves as a unique representation of an anisotropic wavelet packet basis.

According to trees, some operations have to be defined:

- child(S, d, k) returns the sub-band that results from low-pass filtering $(k = 0)$ or high-pass filtering $(k = 1)$ of sub-band S in horizontal $(d = 0)$ or vertical $(d = 1)$ dimension.
- parent(S, d) returns the sub-band T with child$(T, d, k) = S$ for some $k \in \{0, 1\}$.

3. ANISOTROPIC BEST BASIS

Several methods can be used to find the best wavelet packet basis for a given purpose.[13–15] Cost functions can be used that determine a measure of (negative) usability for a given data sequence with respect to a given purpose (cost : $L^2(\mathbf{Z}) \to \mathbf{R}$). The well-known best-basis algorithm[13] for wavelet packet bases is an efficient way to calculate the best basis, i.e. the basis that minimises the overall cost function value in the case of additive cost functions.

However, this algorithm relies on a tree-like decomposition structure. Therefore, it has to be modified to work with bushes. In the isotropic case the algorithm has to decide for each sub-band whether to decompose it or not. In the anisotropic case it can either decompose it horizontally, vertically, both or not at all. To be able to decide this, the best basis of the horizontally and vertically decomposed sub-bands and the minimised cost function values have to be compared against the sub-band's own cost function value. These values have to be determined recursively. The smallest has to be chosen.

```
BestDecomp $(S)$ :=
    if $\mathrm{bestdim}(S) > 0$
        Filter $(S, \mathrm{bestdim}(S))$
        BestDecomp $(\mathrm{child}(S, \mathrm{bestdim}(S), 1))$
        BestDecomp $(\mathrm{child}(S, \mathrm{bestdim}(S), 2))$
```

Figure 2. Best basis decomposition algorithm based on a spanning tree walk. Leaf-vertices are indicated by bestdim(S)=0

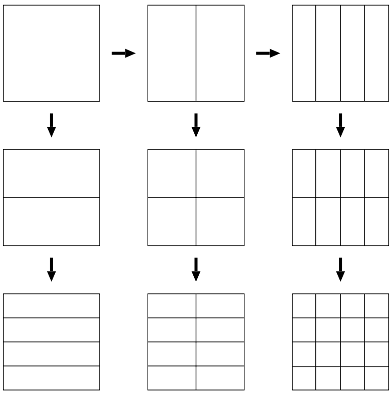


Figure 3. All sub-bands of a full anisotropic wavelet packet decomposition

In the isotropic case it is possible to include the actual calculation of the transform according to the best basis into the determination of the best basis. This avoids a second decomposition run and, thus, saves computation time. However, this is very complicated in the anisotropic case and requires an enormous amount of memory. Additionally, it does not pay because the actual decomposition has a lower complexity than the best basis determination (see below). Therefore, the conventional second-run approach is chosen here.

- First, we have to calculate the cost for each sub-band. This is done by generating all sub-bands as in Figure 3 and calculating the cost for each sub-band immediately after it is created.
- Then, we determine for each sub-band the best cost and the best decomposition dimension by:

$$\begin{aligned}
\mathrm{bestcost}(S,k) &:= \begin{cases} \mathrm{cost}(S) & k=0 \\ \mathrm{bestcost}(\mathrm{child}(S,k,1)) + & \\ \quad \mathrm{bestcost}(\mathrm{child}(S,k,2)) & k=1\dots n \end{cases} \\
\mathrm{bestdim}(S) &:= \operatorname*{argmin}_{k=0\dots n} \mathrm{bestcost}(S,k) \\
\mathrm{bestcost}(S) &:= \mathrm{bestcost}(S, \mathrm{bestdim}(S))
\end{aligned}$$

  This is a recursive procedure ending at leaf sub-bands (sub-bands at maximum decomposition level) where $\mathrm{bestdim}(S)$ is set to 0 and $\mathrm{bestcost}(S) = \mathrm{bestcost}(S,0) = \mathrm{cost}(S)$.

- The best basis decomposition is then performed based on original data by applying a tree-walk on a spanning tree. This leads to the algorithm in Figure 2.

As opposed to isotropic wavelet packets, performing a full decomposition and calculating all possible sub-bands is not the same task for anisotropic wavelet packets. This is, by the way, a consequence of the fact that bushes are not cycle free. To perform a full decomposition, i.e. calculating all sub-bands with maximum decomposition level in each direction, is just as much work as for isotropic wavelet packets, i.e. $O(n \log n)$. This is because the resulting sub-bands actually contain isotropic wavelet packets. Accordingly, the decomposition to a specific basis as in Figure 2 has the same complexity.

To see how much work it is to calculate all possible sub-bands, look at Figure 3. Again, each coefficient is calculated by the inner product of a filter and another sub-band. So we have to count the number of coefficients. Following the construction of Figure 3 we get

$$\text{complexity} = n((l+1)^d - 1) = O(n(\log n)^d)$$

where n is the data size, d is the dimension ($d = 2$ in our case) and l is the decomposition depth. It is assumed that $l = \log_2 \sqrt[d]{n} - c$. This means that the decomposition depth is increased by 1 if the data size is increased by a factor of 2 in each dimension.

4. ZEROTREE CODING

Zero-tree based algorithms arrange the coefficients of a wavelet transform in a tree-like manner. All coefficients are connected by trees that are rooted in the approximation sub-band's coefficients (the sub-band that is low-pass filtered only). Furthermore, a zero-tree is a sub-tree which entirely consists of insignificant coefficients. The significance of a coefficient is determined by a threshold which plays an imported role in zero-tree coding algorithms. Zero-trees can be viewed as a collection of small coefficients with approximately equal spatial position.

The SMAWZ[4] codec is used throughout this paper. It is very similar to SPIHT. Coefficients are encoded bit-plane-wise. Insignificant coefficients can mostly be grouped in two types of zero-trees, allowing either the root or its direct offspring to be significant, i.e. greater than the bit-plane's threshold. The difference between SMAWZ and SPIHT is that SMAWZ applies a spatially oriented coefficient scan order and has a fixed memory demand because it dispenses with list structures to store the significance state and uses bit-maps (significance maps) instead.

SMAWZ uses arithmetic coding to compress the encoded significance, sign and refinement information further losslessly. Context models are basically taken from EBCOT.[16] As with SPIHT, the resulting bit-stream is rate-scalable.

SMAWZ also is compatible with wavelet packet decompositions. In this case it is much more complicated to build trees of coefficients. Similar sub-bands have to be associated to define parent-child relationships between their coefficients. To do so, sub-bands are connected by trees which we will call similarity trees.[4] Similarity trees are rooted in the approximation sub-band and connect all leaf sub-bands, i.e. all sub-bands that are not decomposed and have to be encoded. The greater the distance from the root sub-band with respect to the similarity tree, the higher should the frequency range of the sub-bands be. This is because the absolute value of the sub-band's coefficients should decrease with this distance.

To build similarity trees for anisotropic wavelet packets is even more complicated than for isotropic wavelet packets. First of all, it should be clear, which result is expected. Since isotropic bases are a subset of anisotropic bases, the rules for anisotropic decompositions should produce the same similarity trees as the isotropic for isotropic decompositions. For true anisotropic decompositions, the result should closely resemble what one would expect for a similar isotropic decomposition.

This is done by dividing the rules for isotropic decompositions into several sub-steps which follow branches in the decomposition bush only in a single dimension. Figure 4 shows these rules. First, the initialisation is simply a connection of the root node to itself. The rules are supposed to substitute the rules for the isotropic case[4] when applied in groups of appropriate order. For instance, the first rule followed by the second (in the leftmost column) applied to the initial state produces the initial state of the isotropic case. However, if the application of one of these rules is not possible because the anisotropic decomposition bush is not complete (not isotropic), only an approximation of the isotropic case can be achieved. For instance, the second rule of the rightmost column might be chosen instead of the first. Note that often more than one rule could be applied. In these cases the first fitting rule should be chosen from the list.

As soon as similarity trees are defined, one can easily construct coefficient trees. Thus, the SMAWZ algorithm can be applied without major modifications.

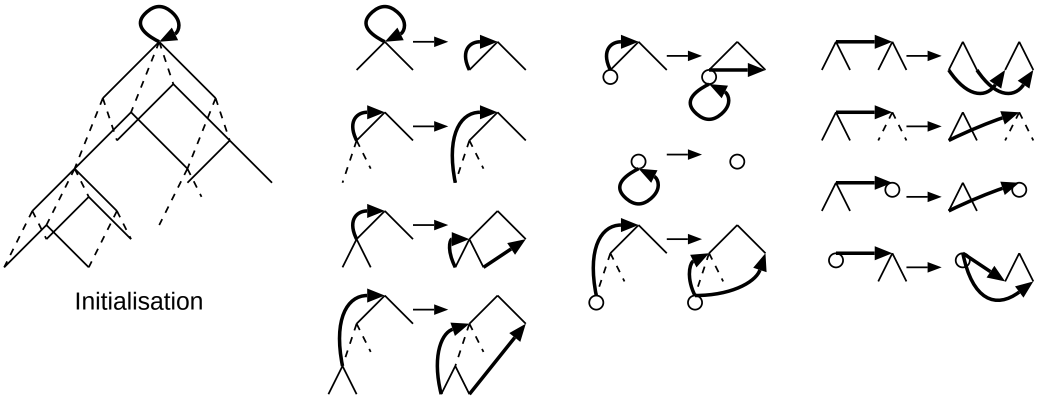


Figure 4. Construction of similarity trees for bushes of anisotropic wavelet packets

5. CODING OF THE ANISOTROPIC DECOMPOSITION STRUCTURE

The decoder can decode bit-streams created by encoders described above only if it is capable of constructing decomposition trees and similarity trees exactly as used by the encoder. The similarity trees are constructed from the decomposition structure which should be known by the decoder. However, the decomposition structure has to be encoded to be available to the decoder.

For pyramidal wavelet decompositions it is sufficient for the decoder to know the decomposition depth. This value can either be set to a fixed number or encoded into the bit-stream. However, the decomposition structure of wavelet packets is more complicated and can consume a serious amount of bit-rate when it is encoded. Therefore, the encoding has to be efficient. Moreover, it is important to have an encoding scheme that is especially efficient on more regular decomposition structures. In the isotropic case one bit has to be encoded for each node, indicating whether the decomposition tree branches at this node. If the maximum decomposition depth is known, bits at this level are always zero and can be omitted. Arithmetic coding can be used to further compress this information by grouping the bits of four neighbouring nodes to a single symbol.

Again, the anisotropic case is more complicated. To encode a decomposition structure optimally (with a minimum number of bits), it is essential to start with a unique representation of this structure. Otherwise, the encoded bits necessarily contain redundancy. Fortunately, a bush *is* a unique representation of the decomposition structure. However, this does not mean that it is trivial to encode this structure without redundancy. The algorithm in Figure 5 does this.

It takes into account that branches of a node imply certain branches of descendant nodes: Firstly, if the bush branches at a node in dimension d, then all child nodes in any dimension different from d have to branch in dimension d. Secondly, if the bush does not branch at a node in dimension d, then at least one child node in a dimension different from d does not branch in dimension d. The algorithm, therefore, keeps track of dimensions in which nodes must or must not branch ("dobranch" and "nobranch") and does not encode this information. This removes all redundancy from the bush's structure.

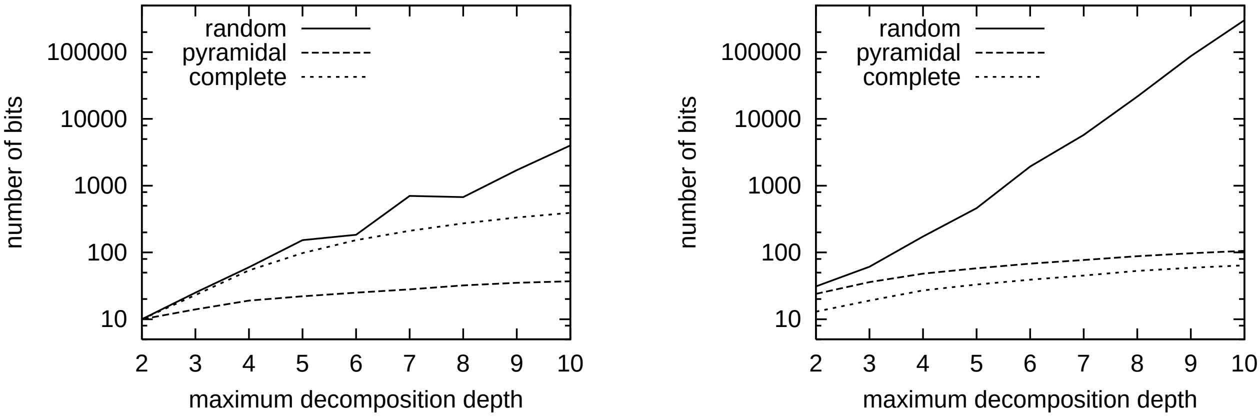
Additionally, nothing has to be encoded if the maximum decomposition depth is reached (if it is known). At one point, two Boolean values are encoded together where only three states are possible. To encode this without redundancy, an arithmetic coder is necessary. This coder also exploits probabilistic redundancies in the structure.

Figure 6 shows the coding results compared to the isotropic case. Note that in both cases regular structures such as the pyramidal and the complete decomposition can be encoded very efficiently. Higher bit-rates for the anisotropic case are a consequence of the fact that the number of possible bushes is much higher than the number of trees with the same maximum depth.

```
Encode ($B$) :=
    Encode (root ($B$), $\emptyset$, $\emptyset$)

Encode ($S$, dobranch, nobranch) :=
    for all dimensions $d$
        if not nobranch[$d$] and not dobranch[$d$]
            Encode (haschildren($d$))
    if a dimension $d$ exists with haschildren($S, d$)
        for all dimensions $e$
            $D_{1,2}[e] \leftarrow$ haschildren($S, e$) $\wedge\, d \neq e$
        for all dimensions $e$ with $\neg$ haschildren($S, e$)
            $D_1[e] \leftarrow$ haschildren(child($S, d, 1$), $e$)
            $D_2[e] \leftarrow$ haschildren(child($S, d, 2$), $e$)
            Encode $D_1[e]$ and $D_2[e]$ together (note: at least one is *false*)
            $N_1[e] \leftarrow \neg D_1[e]; N_2[e] \leftarrow \neg D_2[e]$
        Encode (child($S, d, 1$), $D_1$, $N_1$)
        Encode (child($S, d, 2$), $D_2$, $N_2$)
```

Figure 5. Algorithm to encode an anisotropic wavelet packet decomposition structure without redundancy



(a) isotropic WP

(b) anisotropic WP

Figure 6. Number of bits necessary to encode a 2-D isotropic wavelet packet tree structure (a) and a 2-D anisotropic wavelet packet bush structure (b). The numbers for the random decomposition represent the maximum of ten randomly generated decompositions

6. EXPERIMENTAL RESULTS

A selection of grey-scale images with 512×512 pixel and 8 bit per pixel is used for experiments. The type of images ranges from natural images to artificial textures. The compression performance of SMAWZ using the pyramidal wavelet transform as well as the conventional and the anisotropic wavelet packet transform is compared. In both wavelet packet transforms the best-basis algorithm is used to determine the basis for decomposition. Three common cost-functions are used in the best-basis algorithm and the results are compared:

Entropy information cost	$\mathrm{cost}(c) = -\sum_i c_i^2 \log c_i^2$,
Logarithmic energy	$\mathrm{cost}(c) = \sum_i \log c_i^2$,
1-norm	$\mathrm{cost}(c) = \sum_i \lvert c_i \rvert$

where c denotes a set $(c_1, c_2, c_3, \ldots)$ of coefficients.

Figures 7-12 show the coding results for several representative images. The decomposition structures shown are computed with the isotropic an anisotropic best-basis algorithm using the 1-norm as cost function which usually gives the best compression results. Sub-figures (d)-(f) show the compression performance for all three cost functions.

Figure 7 shows a well-known image without special features like repeating patterns. While it is known that the compression behaviour of such an image can hardly be improved by the use of conventional wavelet packets, we see that this is also true for anisotropic wavelet packets, which not a big surprise. Figure 8 shows an image that usually performs better with wavelet packets. However, the textures in this image are not anisotropic. Therefore, anisotropic wavelet packets cannot increase the compression performance. The opposite happens in Figure 9. Because of the perspective, the patterns are more dense in horizontal than in vertical dimension. While isotropic wavelet packets can only gain about 0.2 to 0.4 dB, anisotropic wavelet packets improve the compression by 0.6 to 1 dB in PSNR.

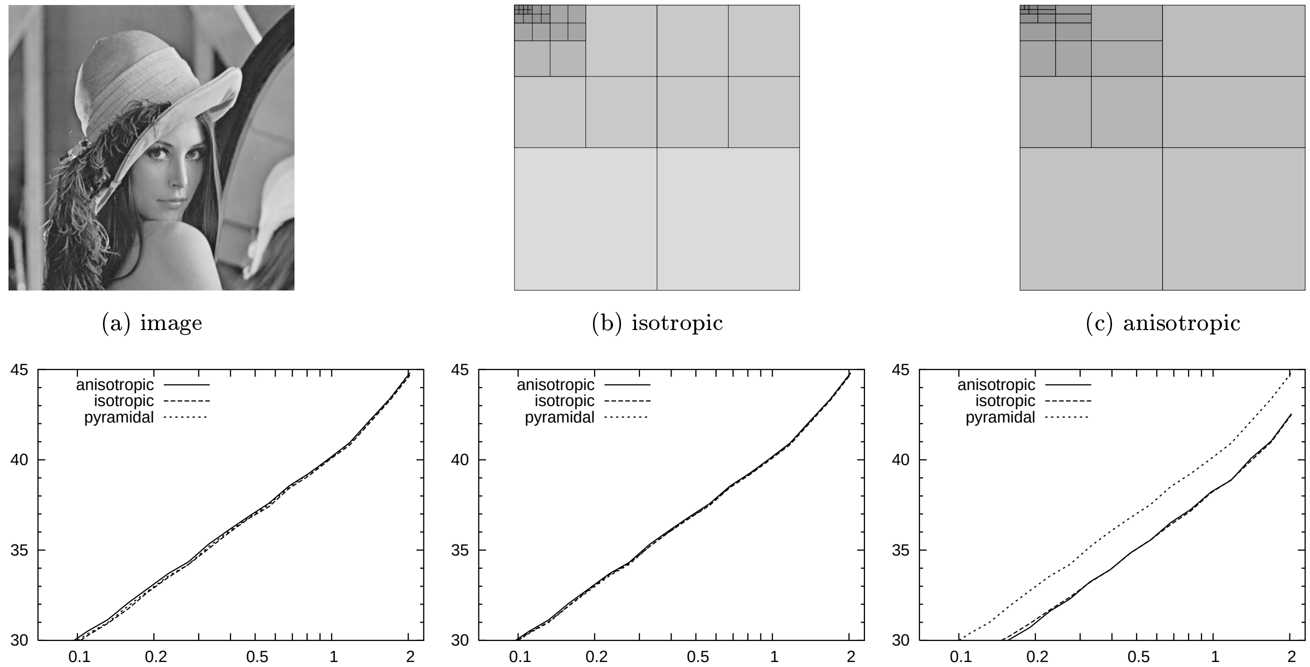
Much better candidates for wavelet packet compression are regular or nearly regular textures. The textures in Figures 10 and 11 are taken from the Brodatz texture library.[17] For the image in Figure 10, the gain is about 1 to 1.4 dB in the isotropic and 1.6 to 2.3 dB in the anisotropic case. Figure 11 shows similar results, although the pyramidal basis performs much worse in this case (up to 13dB). Anisotropic wavelet packets can still improve compression performance by up to 1 dB. Interestingly, this gain is achieved only above about 0.2 bpp. This shows that there indeed are cases where the optimal decomposition basis depends on the targeted bit-rate, which somehow limits the use of cost functions for the determination of optimal bases. Finally, Figure 12 shows that the difference between the isotropic and anisotropic case can be arbitrarily big.

In all these figures we see that the 1-norm and the logarithmic energy cost functions produce nearly equal (and quite optimal) compression results. As was shown several times before, the entropy cost function is far from being optimal. It sometimes even degrades the compression performance with respect to the pyramidal basis.

7. CONCLUSIONS

As a generalisation of the wavelet packet transform, the anisotropic wavelet packet transform offers improved rate-distortion performance of wavelet based compression schemes for certain types of images at the expense of increased computational complexity. Several problems arise when attempting to make wavelet compression algorithms (especially zero-tree algorithms) work with anisotropic wavelet packets, which include: finding a unique representation of the decomposition structure, developing a best-basis algorithm, encoding the decomposition structure without redundancy, and constructing useful coefficient trees. These problems have successfully and efficiently been solved in this paper.

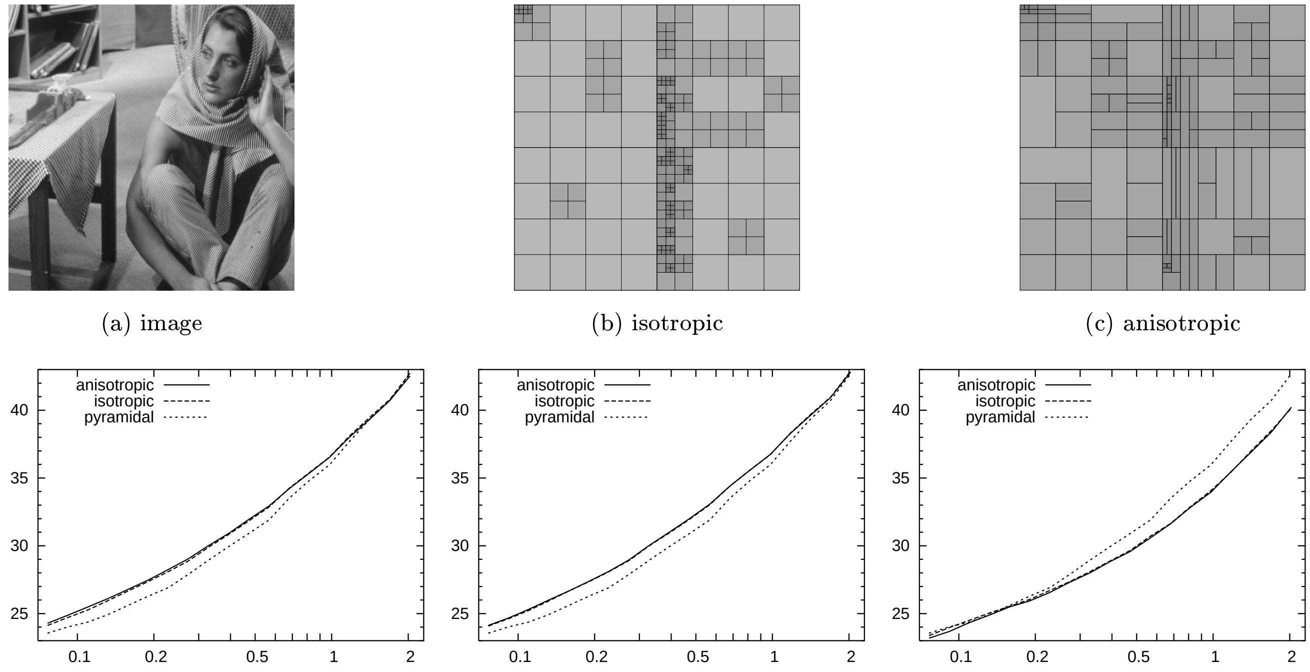
While the computational complexity of the decomposition and encoding for a fixed anisotropic wavelet packet base equals that of isotropic wavelet packet bases (i.e. $O(n \log n)$), the complexity of the best-basis algorithm is increased to $O(n(\log n)^2)$. On the other hand, the compressed image quality can be improved for special images containing regular patterns that have different properties in different dimensions. Compared to isotropic wavelet packets, the gain is up to 0.6 dB for some natural images and up to 1 dB for textures in the experiments conducted for this paper.



(a) image (b) isotropic (c) anisotropic

(d) 1-norm (e) log. energy (f) entropy

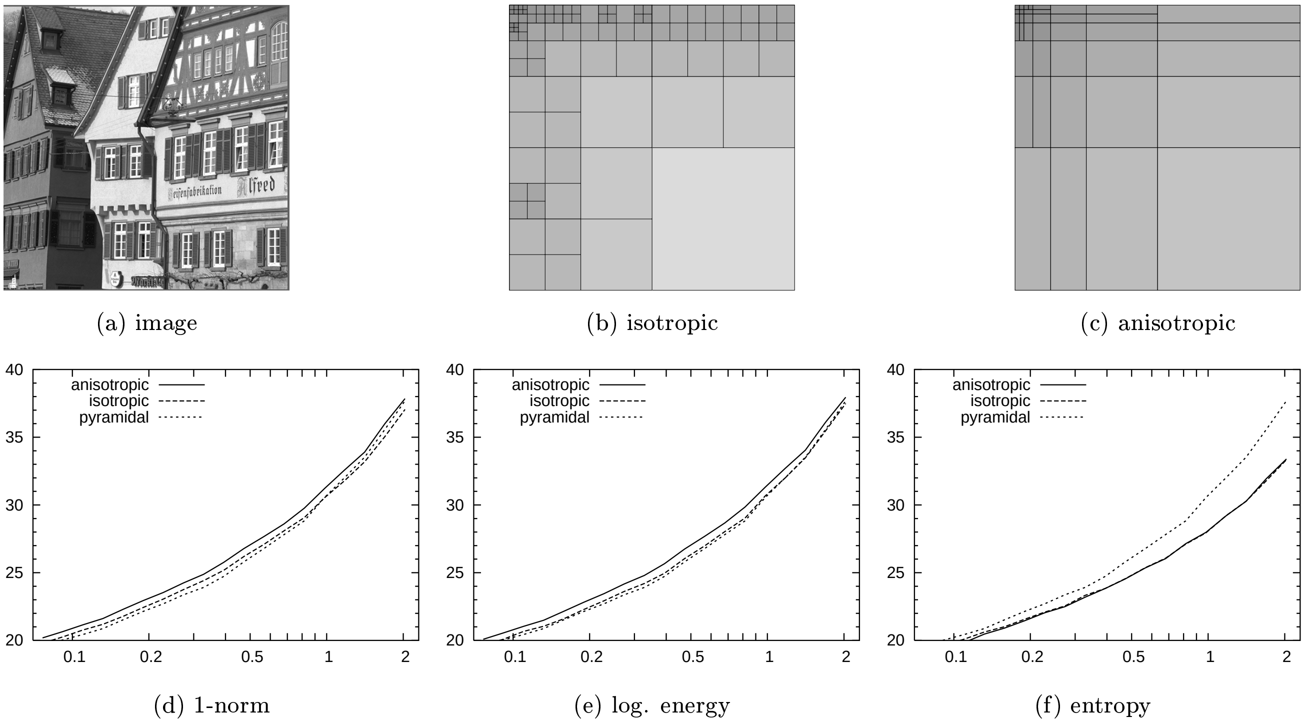
Figure 7. Coding results for image "Lena". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel



(a) image (b) isotropic (c) anisotropic

(d) 1-norm (e) log. energy (f) entropy

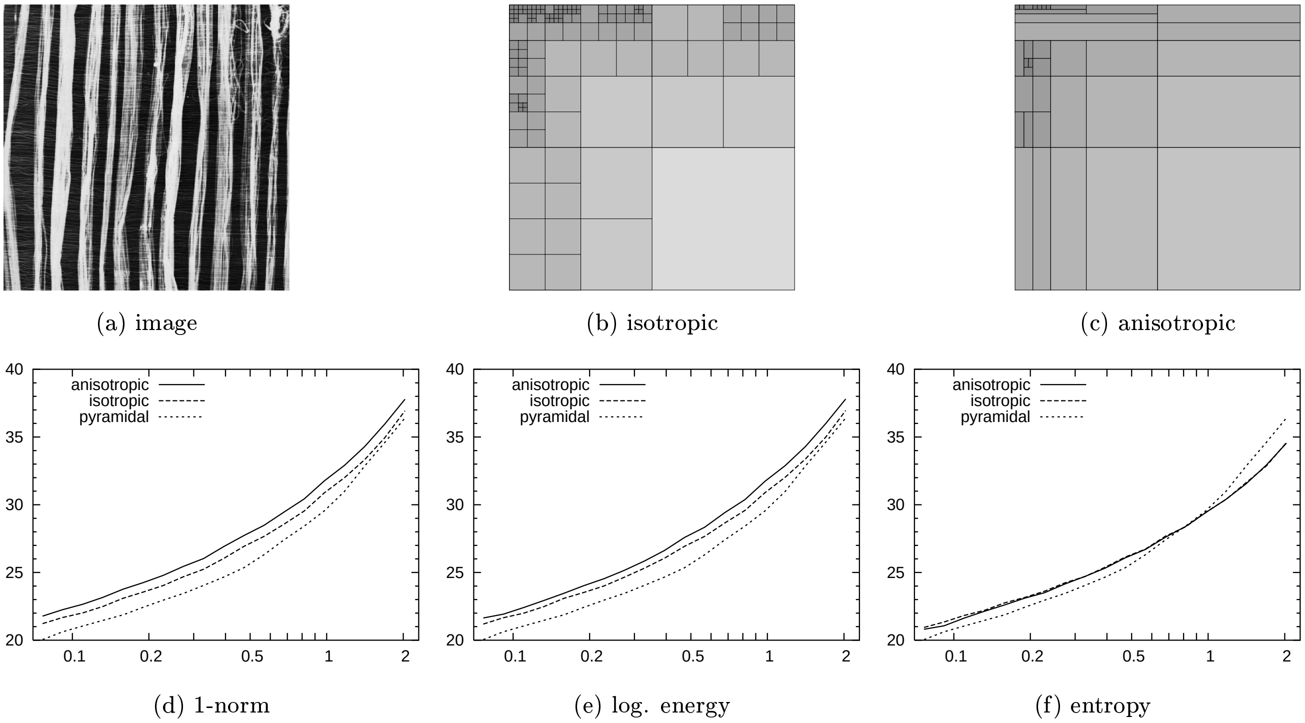
Figure 8. Coding results for image "Barbara". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel



(a) image (b) isotropic (c) anisotropic

(d) 1-norm (e) log. energy (f) entropy

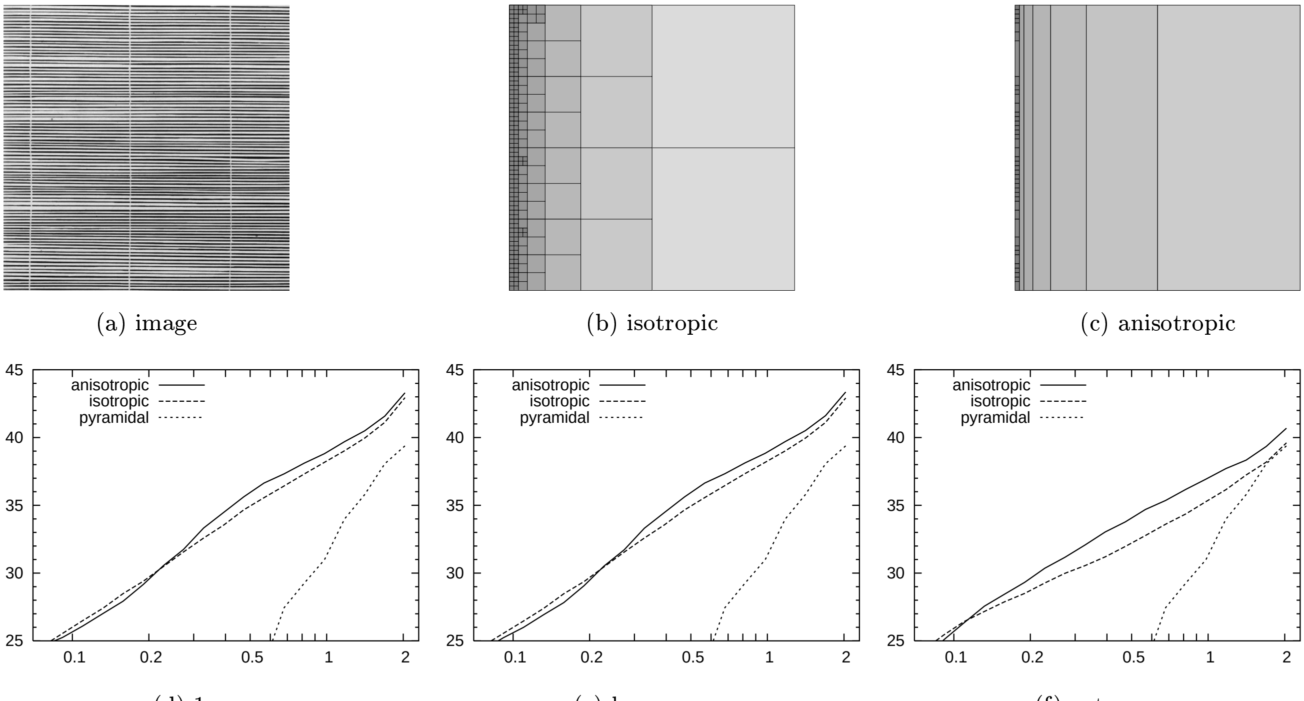
Figure 9. Coding results for image "Houses". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel



(a) image (b) isotropic (c) anisotropic

(d) 1-norm (e) log. energy (f) entropy

Figure 10. Coding results for image "D51". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel



(a) image

(b) isotropic

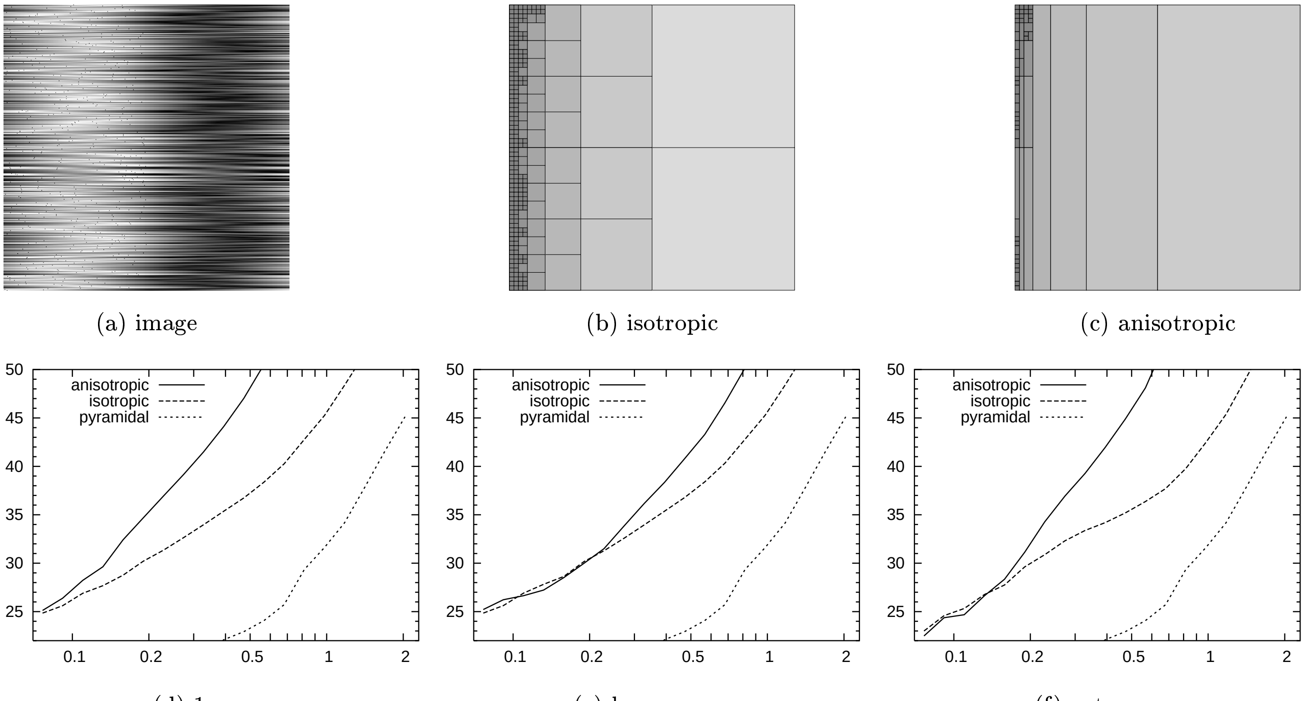
(c) anisotropic

(d) 1-norm

(e) log. energy

(f) entropy

Figure 11. Coding results for image "D49". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel



(a) image

(b) isotropic

(c) anisotropic

(d) 1-norm

(e) log. energy

(f) entropy

Figure 12. Coding results for image "Artificial". Sub-figures (b) and (c) show the best bases according to the 1-norm. (d), (e) and (f) show the PSNR performance depending on the bit-rate in bits per pixel

8. ACKNOWLEDGMENTS

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