Applied and Computational Category Theory (ACCAT): On the Origins and Some Own Work [1]

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It is the objective of this contribution to point to the increasing interdisciplinary importance of category theory and to explain the origins of the acronym ACCAT and to present a brief description of some pieces of own work.
It is well-known that Category Theory (CAT) is a powerful mathematical modeling language and the notion of a category (Cat) is abound in mathematics and various other disciplines. CAT provides far reaching universal construction principles having a wide area of existing and potential future applications, especially in computer science, including the semantical foundations of topics in software science and development. With other words, CAT provides a unifying formal (mathematical) language with constructive elements, of increasing importance in mathematics, computer science and AI and further disciplines (not to forget logic). Due to its generality it is of highly interdisciplinary nature being capable of building links and bridges between very different areas - often providing a common, formal linguistic basis.
The origins of ACCAT, Applied and Computational Category Theory: During my time at RISC-Linz, the Research Institute for Symbolic Computation at the University of Linz (1990 - 1996), I started to give introductory university courses in the frame of the RISC-Linz curriculum with the title "Categories, Fiberings, Sheaves, and Topoi" and "Computational Category Theory (with ML)". The latter title was mainly motivated by the very interesting textbook by *Rydeheard/Burstall: "Computational Category Theory". Prentice Hall (1988)*, where the basic categorical notions and constructions are implemented in ML (i.e. represented as functional programs), thus opening interesting computational aspects. Since I was very interested in CAT applications (inside and outside of mathematics) and in computational aspects I chose the name "ACCAT" for my newly established working group there. With this group I participated in a big European project MEDLAR I (ESPRIT II) and MEDLAR II (Esprit III) where we represented, among others, ACCAT topics. MEDLAR is the acronym for "Mechanizing Deduction in the Logics of Applied Reasoning". During this project I started work on *Categorical Semantics for General Relational Structures*, especially on a General Notion of a Hull Operator for Deductive Systems. (cf. *J.P.: "On a General Notion of a Hull"* in the book *Automated Practical Reasoning, J.Pfalzgraf and D.Wang (Eds.). Texts and Monographs in Symbolic Computation, Springer Verlag Wien-New York, 1995*). For these topics we refer to section 4 and section 3.5 of the ACCAT-Tutorial mentioned subsequently (cf. http://www.cosy.sbg.ac.at/ jpfalz/ACCAT-KI2004.html).
The first ACCAT Tutorial presentation took place in September 1997, where a working group of Professor Wilfried Brauer (Informatik, TU Munich), with whom I was in contact then, asked me to present such a tutorial to them. Some years later, in September 2004, I had the opportunity to organize an own workshop at the 27th German

[1] Abstract of an opening talk given at the 1st ACCAT workshop at European Joint Conferences on Theory and Practice of Software, ETAPS-2006, TU Vienna, March 25 - April 2, 2006, Austria

Conference on Artificial Intelligence, September 20 - 24, University of Ulm, Germany (cf. http://www.cosy.sbg.ac.at/ jpfalz/ACCAT-KI2004.html), where I presented a new and updated tutorial, the ACCAT Tutorial mentioned on our ETAPS-2006 webpage (cf. http://www.cosy.sbg.ac.at/ jpfalz/ETAPS-2006.html).
After reading that ACCAT Tutorial, my colleague Hartmut Ehrig made the proposal to organize a joint ACCAT Workshop as a satellite event at ETAPS-2006, this was accepted by ETAPS. On our ETAPS ACCAT workshop webpage we describe this new cooperation contact with the words: "The organizers are representatives of categorical methods for several areas like Geometry, Neurobiology, Cognitive Sciences, and Artificial Intelligence on one hand and Software Science and Development on the other hand. Categorical methods are already well-established for the semantical foundation of type theory (cartesian closed categories), data type specification frameworks (institutions) and graph transformation (adhesive high level replacement categories), which are most relevant for ETAPS."
Our ACCAT group at RISC-Linz participated as an associated member in the *European TEMPUS project* on "Applications of Categories in Computer Science", invited by my Czech colleagues Jiří Adámek and Jiří Rosicky. Furthermore, we were invited by MEDLAR partners to participate in the *European COST Action 15* on "Many-valued Logics for Computer Science Applications", where I represented my work on Logical Fiberings - a recent publication dealing with that topic is *J.Pfalzgraf: On Logical Fiberings and Automated Deduction in Many-valued Logics Using Gröbner Bases. Invited Article. Revista Real Academia de Ciencias, Serie A de Matemáticas, RACSAM, Vol. 98(1), 2004. (Royal Academy of Sciences of Spain). Special issue on "Symbolic Computation in Logic and Artificial Intelligence". Editors: Luis M. Laita, Jose Antonio Alonso, Eugenio Roanes-Lozano.* Logical Fiberings provide systems of distributed logics for Multiagent Systems (MAS) and cooperating robots.
Later, when I was contacted by "AUI" (the Austro-Ukrainian Institute for Science and Technology), I heard that our ACCAT group was the only one of that type in the country dealing with category theory and applications. We were invited by a group of mathematical physicists to cooperate with them. They were interested in categories in mathematical physics – actually, category theory appears in quantum theory. So, for example, one can read in the article by *Joyal and Street (in the Springer Lecture Notes in Mathematics, Vol.1488 (1990): "Category Theory"*) on page 414 a remark about the connection between knot theory, Feynman diagrams, category theory, and quantum groups. AUI invited me to send an article to the special issue of Ukrainian Journal of Physics, dedicated to the 70th birthday of Prof.Walter Thirring (cf. *J.Pfalzgraf: On a category of geometric spaces and geometries induced by group actions. Ukrain. Journ. Physics 43,7 (1998) 847-856*).
We conclude this period of work with a brief report on theses work done in my ACCAT group at RISC-Linz. With the help of category theory as a unifying language, Karel Stokkermans, in his dissertation, was able to solve an older problem, raised by Bruno Buchberger. The basic question was whether there is a common formal basis for the notion and method of *Gröbner Bases (Buchberger Algorithm), Resolution, and Critical Pair Completion (Knuth-Bendix).* Category theory helped to find a satisfactory natural answer. His results are published in the Journal of Symbolic Computation (1999).
A substantial part of technical work in CAT is to check whether a given diagram of morphisms (arrows) is commutative. I proposed this problem to Wolfgang Gehrke: to develop a method to check commutativity of diagrams automatically (with the help of a computer). Of course, one has to restrict the study of this problem to "tractable" cases, since the general problem certainly will be undecidable. He concentrated his studies

on very interesting (and for computer science important) classes of categories, namely *monads* and *comonads*. Translating the original question into a corresponding problem of equational rewriting resulted in a decidable solution.
In the framework of a cooperation with other scientists in Linz and Spain, Josef Schicho learned about problems around the proof of a result called a theorem of Fried and MacRae, where specific algebraic proofs had been given so far. He was able to translate the corresponding notions and problems into the language of category theory and thus was able to find a general solution to the original problems (published in the international journal Archiv der Mathematik (1995)).
Viorica Sofronie-Stokkermans worked very successfully with categorical language and sheaves to obtain general semantical models in the field of systems of cooperating agents (especially applicable to cooperating robots).

In the end of 1996, I started my work at the University of Salzburg with the intention to continue ACCAT work there, among others.
Connectionism, Artificial Neural Networks (ANN) are very interesting, interdisciplinary and challenging fields. I started work on ANN topics during my time in a high-tech company in Munich, mainly concentrating on industrial applications using ANN computer simulations. Later, I started to work on theoretical topics. We give a brief report of work on: A Category of Geometric Nets and Artificial Neural Network Structure Modeling (links to Theoretical Neurobiology).
During a long cooperation period with H.Geiger (a specialist in ANN applications) it turned out that the net structures which he is permanently using can be mathematically modeled in the sense of a *geometric net theory* which we could introduce, based on so-called *noncommutative geometric spaces* - they form the category (**NCG**). That rather new geometric discipline (introduced by Johannes André in the 1970ies) was a large part of my work done in "pure" mathematics. I found a new general approach to model André's original notion of a geometric space (a brief summary of the basic notions can be found in [CASYS-2003] mentioned below). Actually, these geometric aspects of ANN structure modeling formed the interface to category theory in the following sense. To an ANN we can associate a geometric net which is induced by a corresponding geometric space. Geometric spaces form the category NCG, thus we are motivated to introduce a *category of geometric nets*, **GeoNET**. But what is then the basic feature of ANN theory, namely "LEARNING"? It turns out that a LEARNING STEP can be interpreted as a corresponding MORPHISM in the category GeoNET. This motivates us to speak about a novel concept called "homomorphic learning".
In the course of the previously mentioned cooperation we made an interesting experience: The exploitation of this mathematical, categorical approach for modeling the ANNs in a concrete simulation (which H.Geiger applied in an industrial project on optical quality control in his company) led to surprising results and effects. The *economy of the mathematical model* became visible in a considerable increase of performance in the whole computer simulation that even could be measured in terms of concrete *reductions of the overall project costs* ! Thus, Cats can help to save money.
Subsequently, we make some comments on the links to Theoretical Neurobioloy. In December 2002, I obtained an invitation to give a plenary lecture at the "International Conference on Theoretical Neurobiology", February 24 - 26, 2003, New Delhi, India. Organized by National Brain Research Centre, India. The talk I intended to present there had the title "Modeling Connectionist networks: Categorical, Geometric Aspects (towards 'Homomorphic Learning')". I could not attend this conference, but I was invited to give a talk

on this topic (same title) at the International Conference on Computing Anticipatory Systems CASYS'2003, Liège, Belgium, August 11-16, 2003 - (cf. [CASYS-2003]: *J.Pfalzgraf: Modeling connectionist networks: categorical, geometric aspects (towards "homomorphic learning"). Proceedings CASYS'2003, "Computing Anticipatory Systems: CASYS 2003". August 11-16, 2003, Liège, Belgium. American Institute of Physics, AIP Conference Proceedings, Vol.718 (2004), D.M.Dubois (Ed.).* This contribution received a Best Paper Award.
It is interesting to read the announcement of the New Delhi Conference on Theoretical Neurobiology. "Scope of the Conference": Algebra, Geometry, and Logic of Cognition. Artificial and Natural Intelligence. Cognitive Neuroscience. Computational Neuroscience. Dynamical Systems Theory. Functional Imaging of Brain. Neural Correlates of Consciousness. Neural Networks. Neuroinformatics. Neuropsychiatric Disorders.
"Aims of the Conference": ..... One of the main goals of the conference is to provide a platform for experimental and computational neuroscientists to closely interact and exchange ideas with mathematicians working in the areas of category theory and higher dimensional algebra and explore the potential of these sophisticated mathematical methods, in view of their success in solving problems hitherto intractable within the point set theoretic framework, to meet the demands of cognitive neuroscience data. These interactions will also inspire mathematicians to develop new formal tools and techniques tailor-made to suit the unique nature of the brain and thereby accelerate the development of a comprehensive theory of brain function that provides a scientific account of not only photons and action potentials but also of percept, thoughts, emotions, intention, and action.

Concluding, some remarks on recent work follow concerning a general, generic model of a Multiagent System (MAS) based on CAT as unifying mathematical modeling language. Generally spoken, a MAS consists of agents that communicate and cooperate. The CAT modeling approach consequently defines the category **MAS** in such a way that the objects are the (typed) agents (it is necessary to introduce types of agents). The (typed) morphisms are all kind of relations (note that a relation is visualized by a corresponding arrow, thus morphism). It is a hope that categorical construction principles can be deployed to construct MAS scenarios. I presented first steps at my Symposium "Multiagent Systems, Robotics and Cybernetics" at the *17th International Conference on Systems Research, Informatics and Cybernetics (InterSymp-2005), August 1-7, 2005, Baden-Baden, Germany* - cf. http://www.cosy.sbg.ac.at/∼jpfalz/InterSymp-2005.html. Among others, Hartmut Ehrig and Ulrike Prange presented some of their work dealing with CAT notions and problems.
In a diploma thesis finished last year the notions of MAS, Process Algebra (PA) and Petri Net were used as a theoretical basis to model a query and answer system for an application in eTourism. This system is a MAS and can be interpreted as an instance of a Decision Support System (DSS). The diploma student worked with the categorical formulation of MAS, PA and Petri Net and verified the close correspondence between PA and Petri Nets (*Renate Tippler: MAS, Petri Nets, and Process Algebras: Categorical Aspects and Applications. Diploma Thesis, Salzburg, May 2005*).

Finally, we point here to the text included at the end of my ACCAT-Tutorial dealing with the very interesting remarks by Robert Hermann in his book *Geometric Computing Science: First Steps. Interdisciplinary Mathematics, Vol.25 (1991), Math Sci Press*, where he propagates and promotes CAT as an "essential intellectual tool" in the area of Computer Science and Artificial Intelligence.