

Formale Systeme Proseminar

Tasks for Week 6: 12.11.20

The first task remained from last time.

Task 5 Check with a calculation whether the following abstract propositions are equivalent:

- (a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$
- (b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$

Task 2 Show with calculations that for arbitrary sets A and B , we have

$$A \cup (A^c \cap B) = A \cup B.$$

Task 3 Prove with a calculation that the following propositions are equivalent:

- (a) $x \in A \cup (A \cap B)$ and $x \in A \cup (B \cap B^c)$
- (b) $x \in A \cap (B \cup A^c)^c$ and $x \in B^c \cap A \cap (A \cup A^c)$.

Task 4 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

- (a) $P \vee Q$ and $P \wedge Q$
- (b) P and $\neg(P \vee Q)$
- (c) P and $\neg(P \Rightarrow Q)$

Task 5 Are the following statements valid? Why?

- (a) If $P \stackrel{val}{\models} Q$ and $Q \stackrel{val}{\models} R$ and $R \stackrel{val}{\models} S$, then $P \stackrel{val}{\models} S$.
- (b) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then $Q \stackrel{val}{\models} R$.
- (c) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then Q and R are incomparable.

Task 6 Show with a calculation:

- (a) $P \Rightarrow Q \stackrel{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$
- (b) $\neg(P \Rightarrow \neg Q) \stackrel{val}{\models} (P \vee R) \wedge Q$