Formale Systeme Proseminar

Tasks for Week 5, 5.11.2020

Task 1 Prove that:

(a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$

(b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$

(c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Task 2 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

(a) $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$

(b) $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$

(c) $P \vee (P \wedge Q) \stackrel{val}{=} P$

(d) $P \wedge (P \vee Q) \stackrel{val}{=} P$

(e) $P \Rightarrow \neg Q \stackrel{val}{=} \neg(P \wedge Q)$

Task 3 Show with a calculation that the following formulas are tautologies

(a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

(b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 4 Show with calculations that for arbitrary sets A and B, we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 5 Check with a calculation whether the following abstract propositions are equivalent:

(a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$

(b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$