Formale Systeme Proseminar

Tasks for Week 3 - October 22, 2020

Task 1 We are given 3 piles of chocolate. The first consists of 4 bars, the second of 6 bars, and the third of 14 bars. The piles should be evened, so that each pile consists of 8 bars. In each step one may only move chocolate bars from one pile to another. In addition, in one step one may only move n bars from pile x to pile y, if before the move bar y contained exactly n bars. Model the problem as in the example considered in class.

Task 2 Model a simple coffee&tea vending machine with three buttons (for choosing coffee, tea, or canceling an operation) and a socket for inserting coins. You may assume that there exists a single admissible coin (e.g. 1 EUR) and every drink costs the same. Hence, no money exchange happens. Describe the relevant objects being modelled and the choices made in your design of the machine.

Task 3 Consider the following sets:

$A = \{a, b, c, d, e, f\}$,
$B = \{a, c, e, f\}$,
$C = \{b, d, g, h\}$,
$D = \{c, a, f, e\}$.

1. Construct the intersection of any two of the given sets[1].
2. Construct the union of any two of the given sets.
3. Which sets are disjoint, which are subsets of another set, which are proper subsets of another set?

Task 4 Consider the set $S_{\mathbb{N}} = \mathcal{P}(\mathbb{N})$.

1. Write down 3 elements of $S_{\mathbb{N}}$ that are finite sets.
2. Write down an element of $S_{\mathbb{N}}$ that is an infinite set.
3. Find two disjoint subsets of natural numbers, i.e., elements of $S_{\mathbb{N}}$ whose union equals $\mathbb{N}$.

Task 5 Let $A = \{a, b, c, d\}$, $B = A \setminus \{d\}$, and $C = A \cap \{a\}$. Calculate $\mathcal{P}((A \cap B) \setminus C)$ and explain every step.

Please don't forget to turn the page!

[1] Here, any means every!

Task 6 Prove that for any sets X and Y, we have $X \cap Y \subseteq X$.

Task 7 Prove that for any set X, we have $X \cup X = X$.

Task 8 Prove that for any set X there exist sets Y and Z such that $X = Y \cup Z$.

Task 9 Prove that $\emptyset \subseteq X$ for any set X.