# Formale Systeme Proseminar

## Tasks for Week 11, 17.12.2020

**Task 1** Let $A = \{1, 2, 3, 4\}$ and consider the relation

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 4), (4, 3)\}.$$

Show that $R$ is an equivalence relation.

**Task 2** Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid (xy > 0) \text{ or } x = y = 0\}.$$

Prove that $R$ is an equivalence and write down its equivalence classes.

**Task 3** Describe the equivalence classes of the equivalence $\equiv_5$ on $\mathbb{Z}$ defined in the lectures. In general, for a fixed natural number $n$, describe the classes of $\equiv_n$. How many classes are there?

**Task 4** Let $A = \{a, b, c, d\}$. For each of the following partitions of $A$ write down the corresponding equivalence:

(a) $\{\{a, b\}, \{c, d\}\}$,

(b) $\{\{a\}, \{b, c, d\}\}$,

(c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

**Task 5** Let $A = \{a, b, c\}$. How many equivalence relations are there on $A$? List them all.

**Task 6** Give an example of an equivalence on $\mathbb{N}$ with

(a) 3 equivalence classes,

(b) 10 equivalence classes,

(c) 100 equivalence classes.

**Task 7** Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n + 1) \mid n \in \mathbb{N}\}.$$

(a) Find the relation $R^2$,

(b) Find the relation $R^3$,

(c) Can you think of a concise way to describe the reflexive and transitive closure relation $R^*$?