Approximate Tree Matching with pq-Grams

Nikolaus Augsten[a], Michael Böhlen, Johann Gamper
DIS - Center for Database and Information Systems
Free University of Bozen-Bolzano, Italy
`www.inf.unibz.it`

[a] Supported by the Municipality of Bozen-Bolzano.

Motivation — Example Data Sources

- We want to **link data items** in **different databases** that correspond to the **same real world object**.
- **Example query:** Who lives in `Braun`'s apartment?

Land Register

LR

id	*num*	*entr*	*apt*	*owner*
91	1	-	1	Maier
91	1	-	2	Rossi
91	1	-	3	Maier
91	2	A	-	Braun
...				
74	3	A	1	Spiro
74	3	A	2	Barducci
74	3	A	3	Costanzi
...				

SLR

id	*street*
139	SIEGESPLATZ
109	GILMWEG
185	P. R. GIULIANI STR.
91	CESARE ABBA STRASSE
165	MUSTERPLATZ
115	ITALIENSTRASSE
259	TELSERDURCHGANG
207	SERNESIDURCHGANG
33	BOZNER BODENWEG
263	TRIESTER STRASSE
262	TRIENTER STRASSE
285	WALTHERPLATZ
266	TURINER STRASSE
...	

Registration Office

RO

resident	*id*	*num*	*entr*	*apt*
Pichler	30	1	-	1
Rieder	30	1	-	3
Fischer	30	2	A	-
Rossi	30	2	B	1
...				
Spiro	120	3	A	1
Barducci	120	3	A	2
Costanzi	120	3	A	3
...				

SRO

id	*street*
30	Giuseppe-Cesare-Abba-Str.
5220	Bozner-Boden-Str.
3000	Hermann-von-Gilm-Str.
3030	Pater-Reginaldo-Giuliani-Str.
3540	Italienallee
4440	Musterplatzl
7180	Raffaello-Sernesi-Galerie
7590	Telsergalerie
7620	Friedensplatz
7650	Turiner Str.
7740	Trienter Str.
7860	Triester Str.
8580	Walther-v.-d.-Vogelweide-Pl.
...	

!

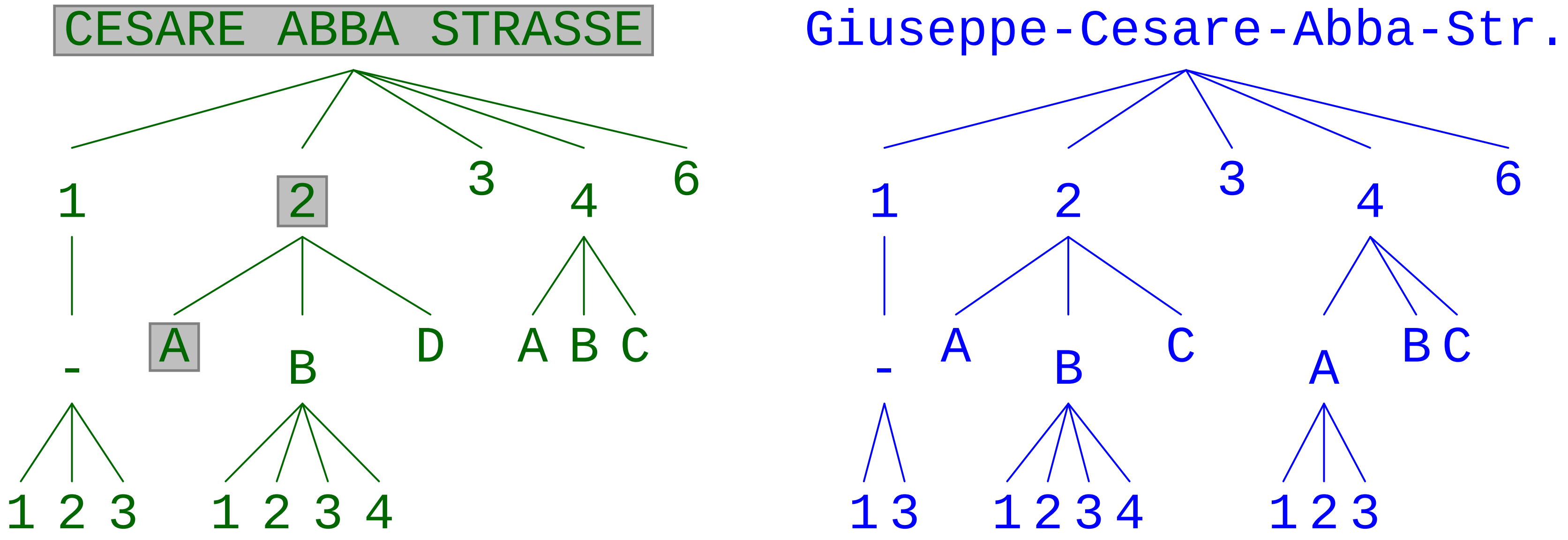
?

Motivation — Address Trees

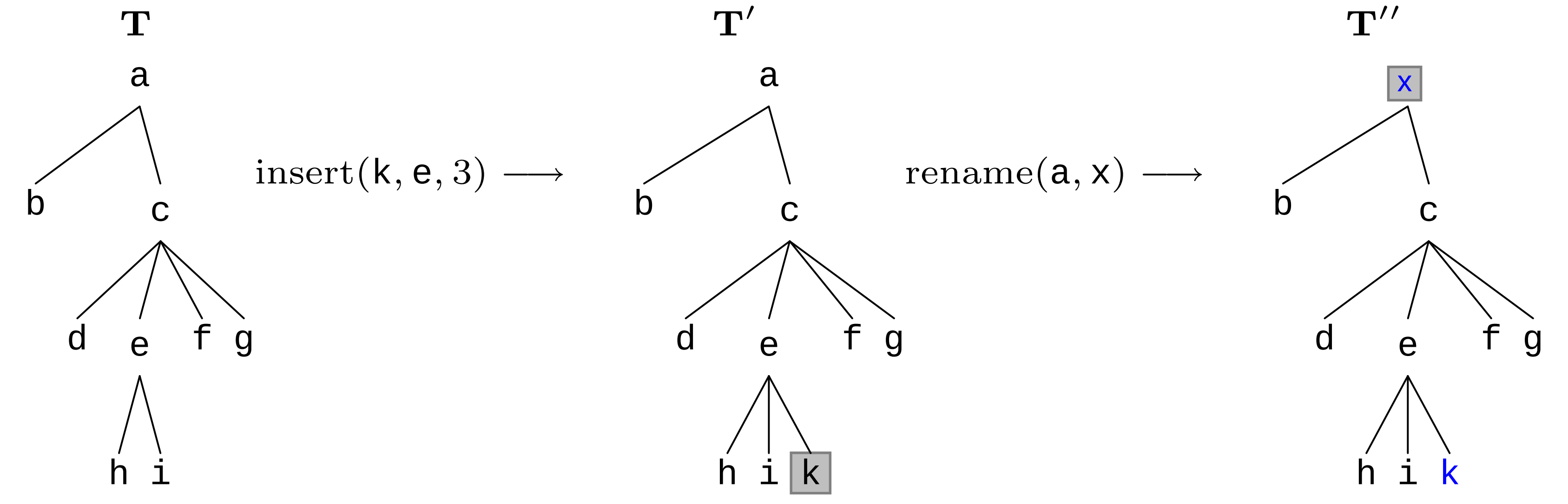
- ☞ residential addresses are hierarchical $\longrightarrow$ **address tree**
- ☞ **Idea:** corresponding streets $\Rightarrow$ similar address tree

How similar are two address trees?

Address trees:



- **Edit distance:** Minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transform one tree into an other.



edit distance: $\text{dist}_{\text{ed}}(\mathbf{T}, \mathbf{T}'') = 2$

- **Problem:** Best algorithms $O(n^2 \log^2(n)) \Rightarrow$ not scalable.

Motivation — Problem Definition

☞ **Our goal:** Find an **efficient** and **effective approximation** of the tree edit distance that

- is **scalable** for large trees,
- emphasizes **structure**.

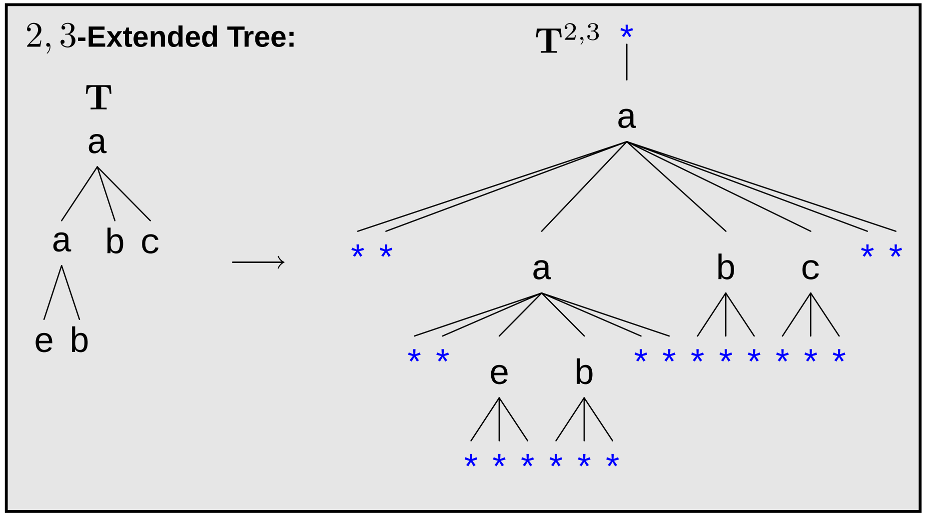
Related Work — Tree Distances

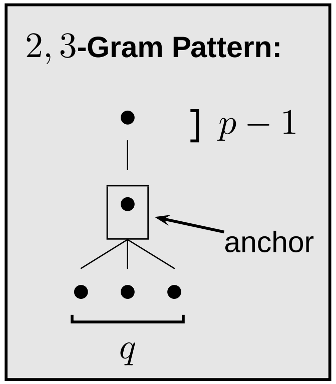
- $n \longrightarrow$ number of tree nodes
- **Tree edit distance:**
  - for balanced trees [Zhang and Shasha, 1989]: $O(n^2 \log^2(n))$
  - for arbitrary trees [Klein, 1998]: $O(n^3 \log(n))$
- **Tree edit distance approximations:**
  - Restricted versions of the tree edit distance:
    - Alignment [Jiang et al., 1995]: $O(n^2)$
    - Isolated subtree [Tanaka and Tanaka, 1988]: $O(n^2)$
    - Top-down [Selkow, 1977, Yang, 1991]: $O(n^2)$
    - Bottom-up [Valiente, 2001]: $O(n) \longrightarrow$ only very specific domains
  - XML versioning [Chawathe et al., 1996, Chawathe and Garcia-Molina, 1997, Lee et al., 2004]: $O(n^2)$ for very different trees
  - Tree-edit distance embedding [Garofalakis and Kumar, 2003, Garofalakis and Kumar, 2005]:
    - $O(n \log n)$
    - guaranteed distance distortion for tree edit distance with subtree move
- **Related work for *strings*:**
  - Navarro [Navarro, 2001]: good overview of the edit distance for strings and its variants
  - Ukkonen [Ukkonen, 1992]: q-grams as lower bound for string edit distance

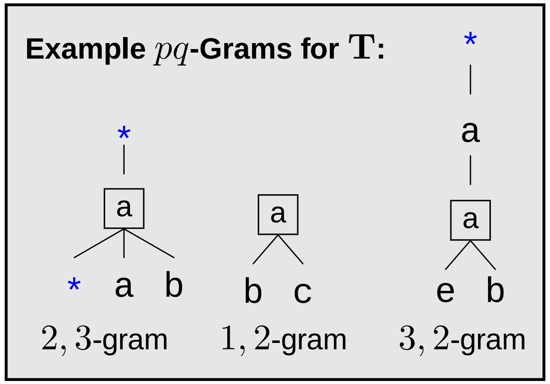
pq-Grams — Subtrees of the pq-Extended Tree

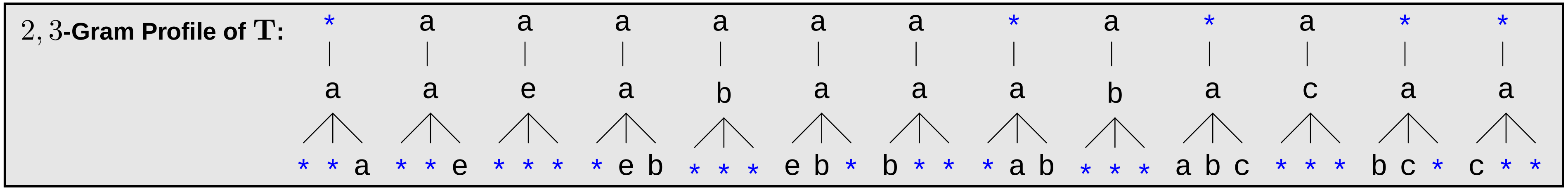
- **Extended Tree $\mathbf{T}^{pq}$:**
  Patch boundaries by adding null nodes (*):
  - $p-1$ ancestors to the root
  - $q-1$ nodes before the first and after the last child of each non-leaf node
  - q children to each leaf
- ***pq*-Gram $\mathbf{G}$:** Subtree of $\mathbf{T}^{pq}$.
  - Anchor node
  - with $p-1$ ancestors
  - and q children.

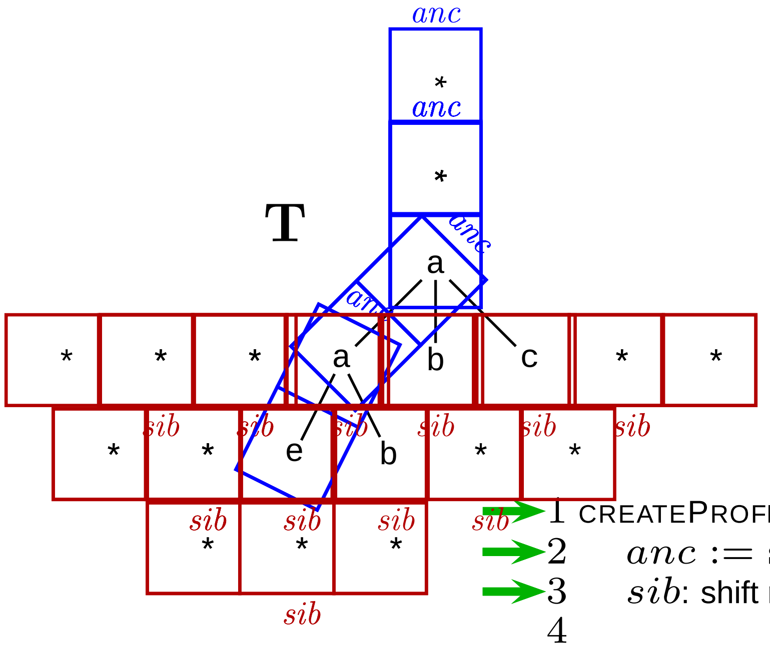
  Contiguous siblings in $\mathbf{G}$ are contiguous siblings in $\mathbf{T}^{pq}$.
- ***pq*-gram Profile $\mathrm{P}^{p,q}(\mathbf{T})$:**
  - Bag of all pq-grams of $\mathbf{T}$.












```
CREATEPROFILE(T, p, q, P, r, anc)
  anc := shift(anc, l(r))
  sib: shift register of size q (initialized with *)

  if r is a leaf then
    P := P ∪ (anc ∘ sib)
  else
    for each child c (from left to right) of r do
      sib := shift(sib, l(c))
      P := P ∪ (anc ∘ sib)
      P := PROFILE(T, p, q, P, c, anc)
    for k := 1 to q − 1
      sib := shift(sib, *)
      P := P ∪ (anc ∘ sib)
  return P
```

P

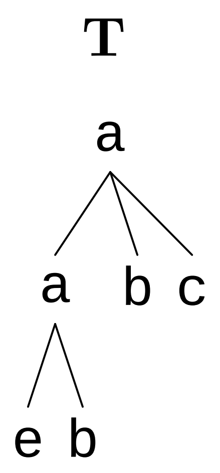
*	a	*	*	a
a	a	*	*	e
a	e	*	*	*
a	a	*	e	b
a	b	*	*	*
a	a	e	b	*
a	a	b	*	*
*	a	*	a	b
a	b	*	*	*
*	a	a	b	c
a	c	*	*	*
*	a	b	c	*
*	a	c	*	*

The pq-gram profile is

- **small** $\rightarrow$ size $O(n)$
- **easy to store**
  - represent the pq-grams by fingerprint **hash value**
  - store profile in **single-attribute relation**
- **allows** effective **distance computation** between trees

Theorem 1 *For tree* $\mathbf{T}$ *with* l *leaves and* i *non-leaves:*

$$|\,\mathrm{P}^{p,q}(\mathbf{T})| = 2l + qi - 1.$$



$\longrightarrow$ $\mathrm{P}^{2,3}(\mathbf{T})$ $\longrightarrow$ $\mathrm{P}^{2,3}(\mathbf{T})$

pq$-gram$	hash
(*,a,*,*,a)	10AE
(a,a,*,*,*)	2F1E
(a,e,*,*,*)	1008
(a,a,*,e,b)	13E1
(a,b,*,*,*)	5F31
(a,a,e,b,*)	AE1D
(a,a,b,*,*)	13DF
(*,a,*,a,b)	F310
(a,b,*,*,*)	5F31
(*,a,a,b,c)	45A1
(a,c,*,*,*)	973F
(*,a,b,c,*)	3F1E
(*,a,c,*,*)	11EF

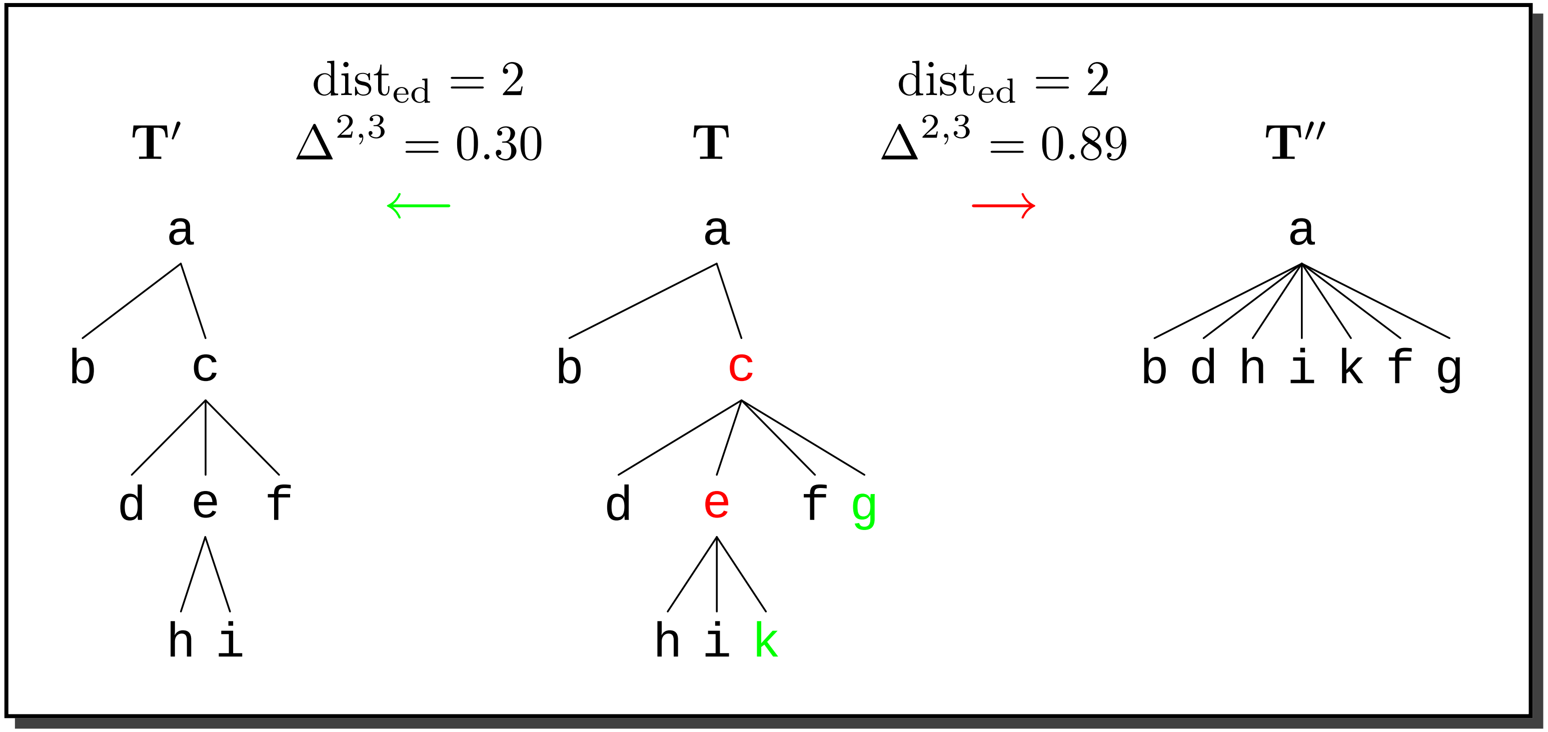
- **Definition 1** *For two trees* $\mathbf{T}_1$ *and* $\mathbf{T}_2$ *the* ***pq*-gram distance** *is:*

$$\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2\frac{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cap \mathrm{P}^{p,q}(\mathbf{T}_2)|}{|\,\mathrm{P}^{p,q}(\mathbf{T}_1) \cup \mathrm{P}^{p,q}(\mathbf{T}_2)|}$$

- can be computed in $O(n \log n)$ time and $O(n)$ space (bag intersection of relations)
- other terms are constants for **normalization**:
  - $\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1$ if trees have no pq-grams in common
  - $\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 0$ if trees have the same pq-gram profile

Properties — Sensitivity to Structure Change

- **Intuition:** Nodes with structural information $\longrightarrow$ more significant
- **Address application:** Mismatch of houses (with subnumbers and apartment numbers) is more significant than mismatch of apartments.



Properties — Sensitive to Structure Change

- ☞ node changes $\Rightarrow$ pq-grams change
- ☞ pq-grams change $\Rightarrow$ distance increases
- ☞ $\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) \approx q + f^p$
  - ➾ **leaf** change: cost depends **only on** $\boldsymbol{q}$
  - ➾ **non-leaf** change: $\boldsymbol{p}$ **prevalent**
  - ➾ $\boldsymbol{p}$ controls structure sensitivity

Theorem 2 *For a complete tree* $\mathbf{T}$ *(fanout* f*, depth* d*) the* **number of** pq**-grams that contain a node** v *of level* l *is:*

$$\mathrm{cnt}_{pq}(\mathbf{T}, \mathsf{v}) = q\,\mathrm{sgn}(l) + \begin{cases} \frac{f^p-1}{f-1}(f+q-1) & \text{if } p \leq d-l \\ \frac{f^{d-l}-1}{f-1}(f+q-1) + f^{d-l} & \text{if } p > d-l. \end{cases}$$

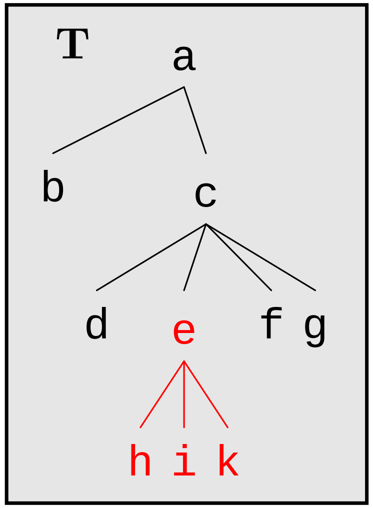
Properties — Robust to Local Changes

- **Intuition:** Weight local changes less than distributed changes!
- **Address application**: missing house with apartment numbers $\longrightarrow$ small difference, but many nodes change
- Local changes $\longrightarrow$ less pq-grams change
  - neighbored nodes “share” pq-grams
  - change counts only once.

Theorem 3 *Delete or update all nodes of subtree with l leaves and i non-leaves $\Rightarrow$ only $2l + iq + q - 1$ pq-grams change*

Example: Delete subtree rooted in e

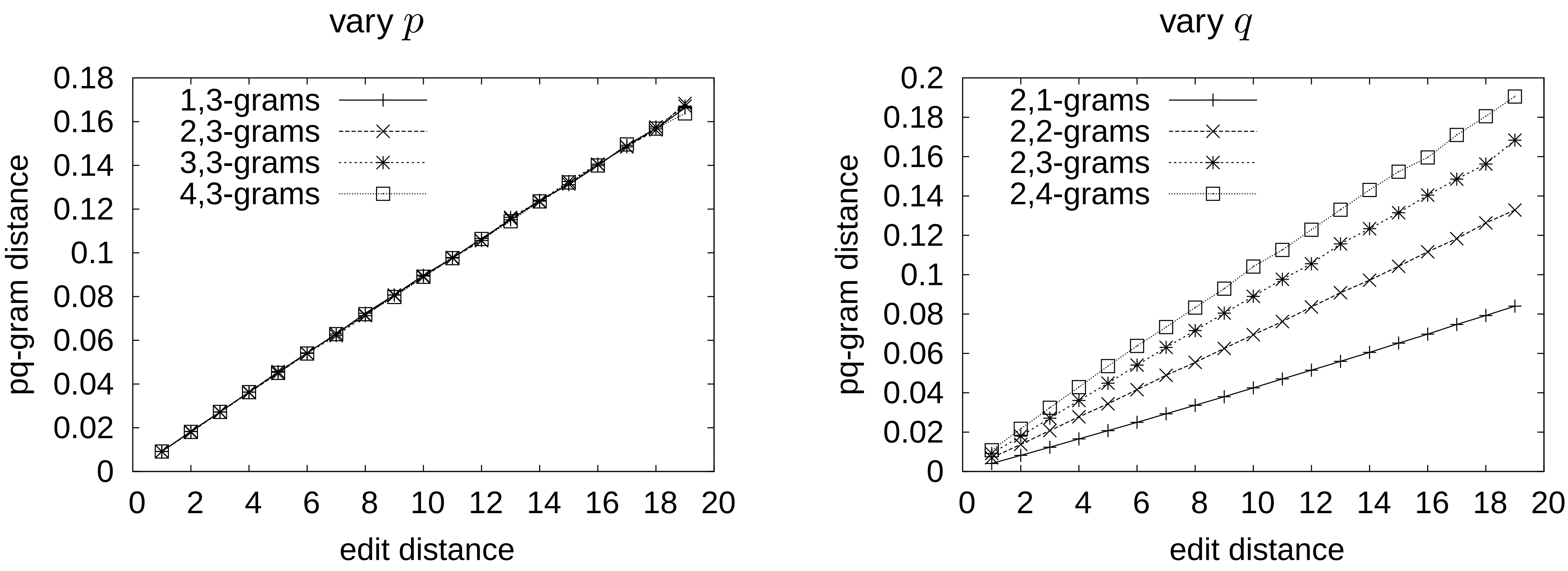
- e $\longrightarrow$ in 11 pq-grams
- h, i, k $\longrightarrow$ in 4 pq-grams each
- actually changing: 11 pq-grams (vs. $3 \times 4 + 11 = 23$)



☞ Cost for **leaf change** $\longrightarrow$ depends only on q

☞ **Experiment:**

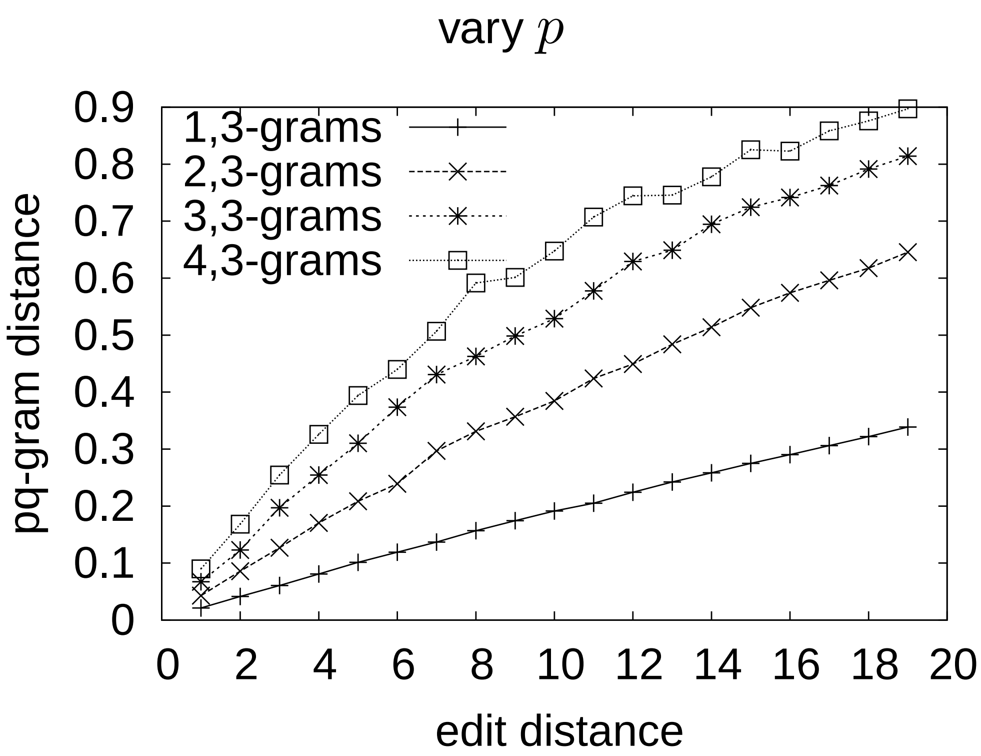
- ➾ delete leaf nodes
- ➾ measure edit distance

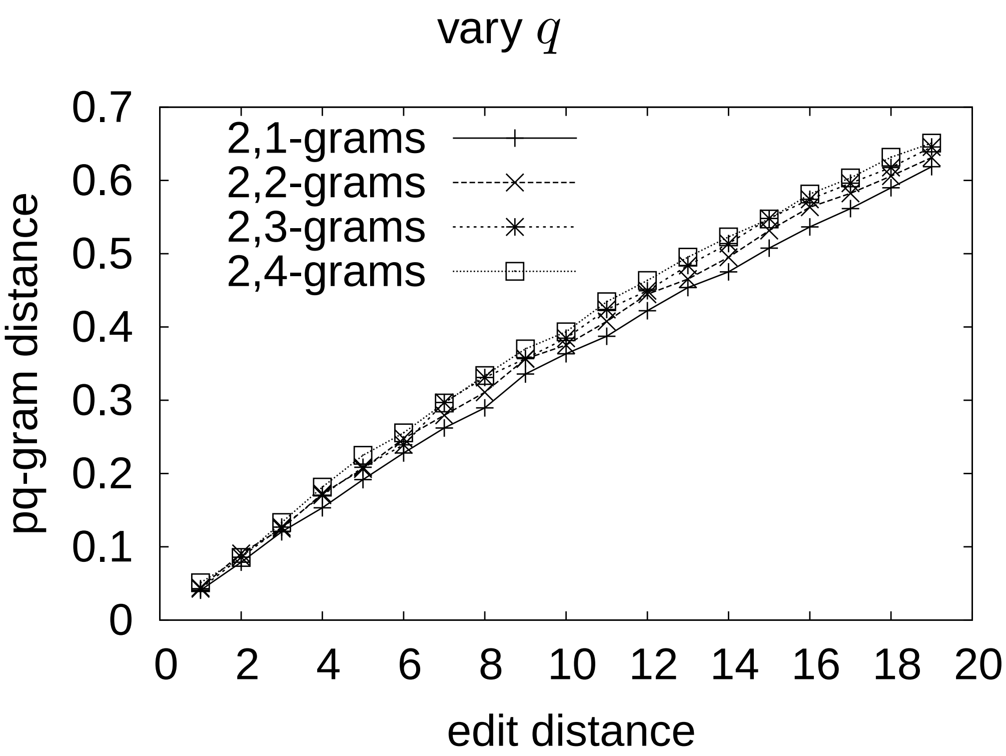


(Artificial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Sensitivity to Structure Changes

- Cost for **non-leaf change** $\rightarrow$ controlled by p
- **Experiment:**
  - delete non-leaf nodes
  - measure edit distance





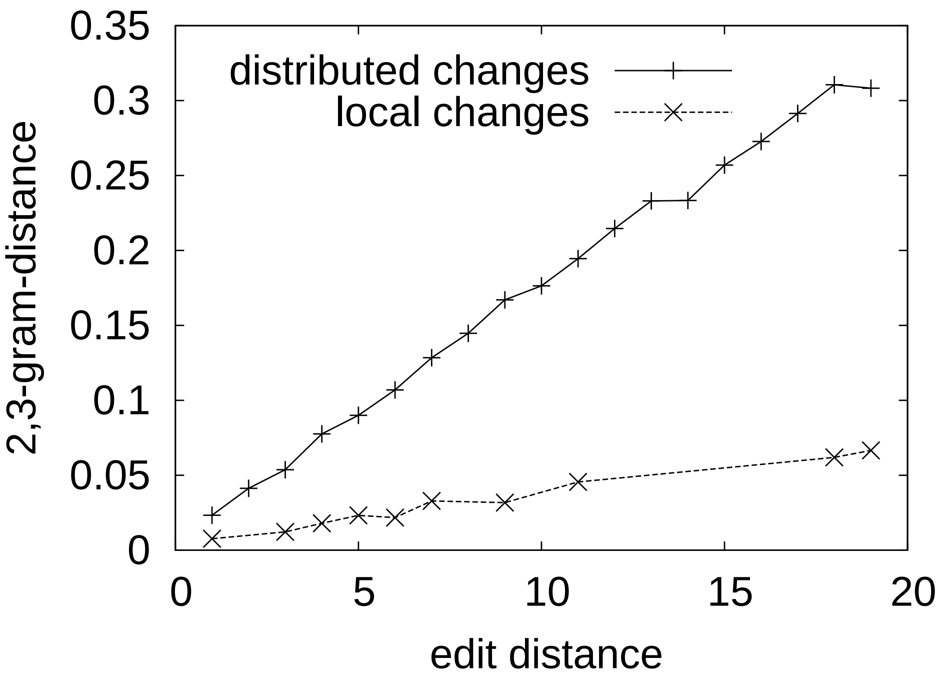
(Artificial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Robustness to Local Changes

☞ **Subtree deletions** → **cheaper** than distributed deletions

☞ **Experiment:**

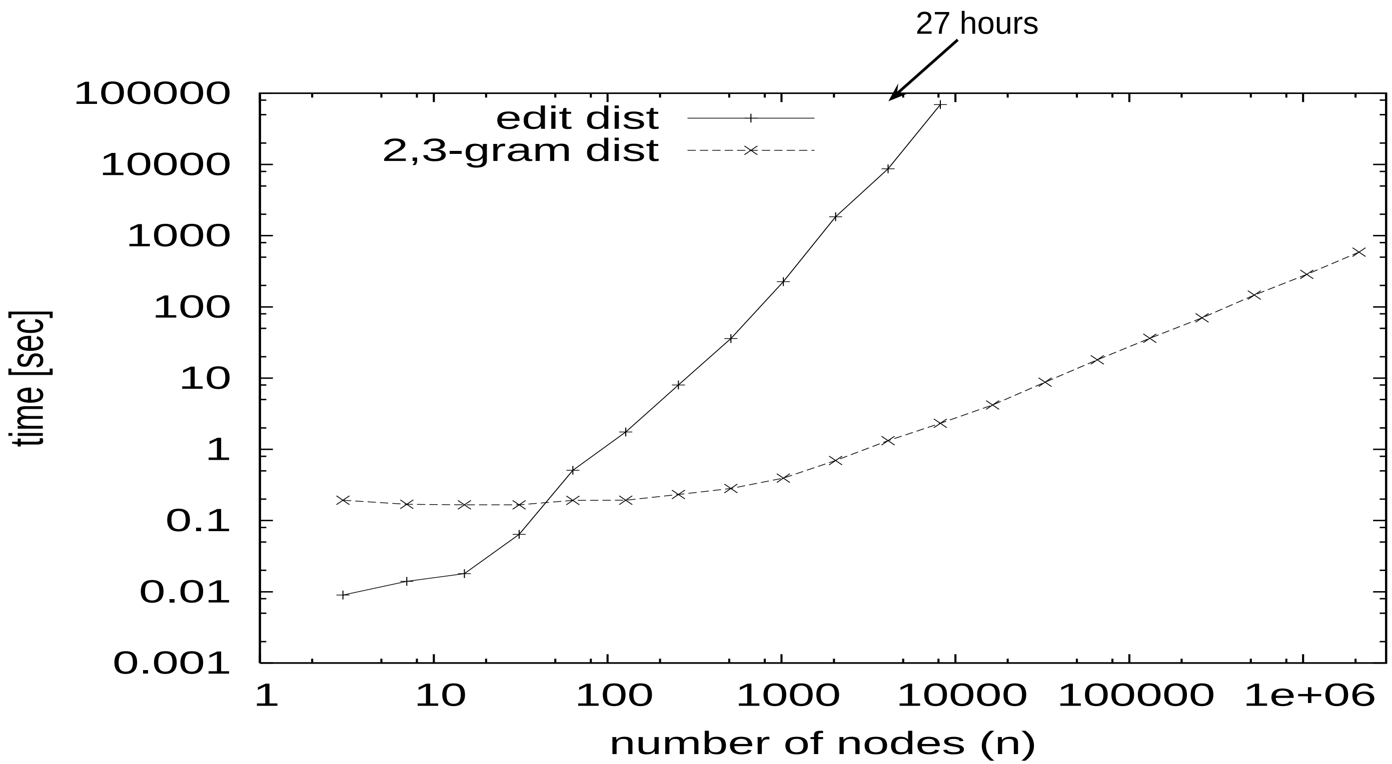
- delete subtree
- randomly delete same number of distributed nodes
- compare edit distance



(Artificial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

Experiments — Scalability to Large Trees

- ☞ pq-gram distance $\longrightarrow$ **scalable** to large trees
- ☞ compare with edit distance
- ☞ **Experiment:** For pair of trees
  - ➾ compute tree edit distance[a] and pq-gram distance
  - ➾ vary tree size: up 2×10^6 nodes
  - ➾ measure wall clock time



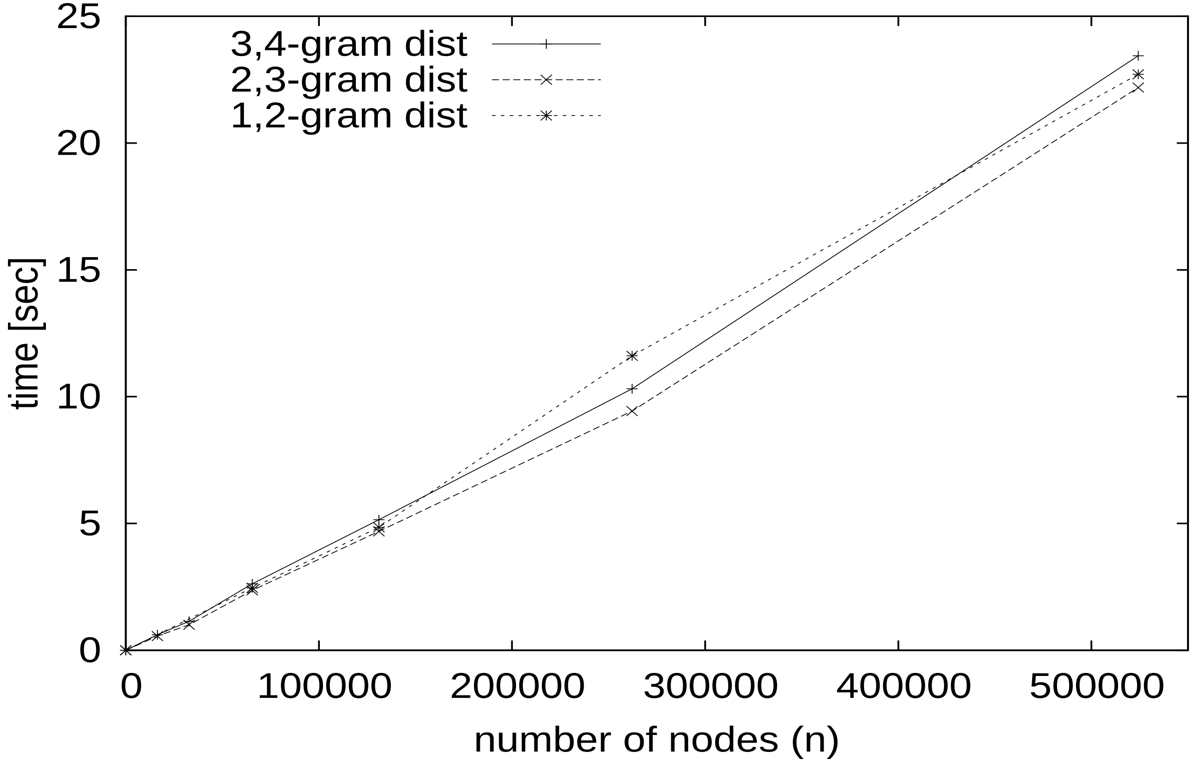
[a] implementation by Zhang and Shasha (`http://www.cs.nyu.edu/cs/faculty/shasha/papers/tree.html`)

Experiments — Influence of p and q on Scalability

☞ Scalability independent of p and q.

☞ **Experiment:** For pair of trees

- compute pq-gram distance for varying p and q
- vary tree size: up 10^6 nodes
- measure wall clock time



Experiments — Effectiveness for Real World Dataset

- pq-gram distance $\longrightarrow$ **effective approximation** of tree edit distance
- test on real world data (**address databases**)
- **Experiment:** For two sets of address trees
  - find matches (closest tree of other set)
  - use different distance functions
  - count correct matches

	accuracy	correct	false pos.	runtime
edit dist	82.7%	248	9	187,538s
1,2-grams	78.3%	235	5	181s
2,3-grams	77.3%	232	4	204s
3,2-grams	79.3%	238	2	180s
tree-embedding	69.0%	207	8	313s
buttom-up	50.0%	150	12	237s

Experiments — Tree-Embedding vs. pq-Grams

tree edit distance embedding	pq-grams
☞ variable shapes	☞ fixed shape
☞ elements can be	
➾ single node (no structure)	
➾ chains (only vertical structure)	☞ horizontal **and** vertical structure
➾ contiguous leaves (only horizontal structure)	
➾ subtrees with vertical and horizontal structure	
☞ guarantees with respect to tree edit distance	☞ emphasizes structure

Typical address tree (nearly complete tree):

Phase	0	1	2	3	4	5	6	tot.
single nodes	29	8	4	2	1	-	-	44
chains	-	1	1	-	-	-	-	2
cont. leaves	-	7	2	-	-	-	-	9
subtrees	-	-	3	4	3	2	1	13

tree-embed	pq-gram
65%	0%
3%	0%
13%	0%
19%	100%

Conclusion and Future Work

- pq-gram distance
  - scalable to large trees
  - emphasizes structure
  - robust to local change
  - effective approximation of tree edit distance
- Ongoing and future work:
  - strict bounds for pq-gram approximations
  - clustering of XML data
  - incremental updates of pq-gram profiles

References

[Chawathe and Garcia-Molina, 1997] Chawathe, S. S. and Garcia-Molina, H. (1997). Meaningful change detection in structured data. In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 26–37, Tucson, Arizona, United States. ACM Press.

[Chawathe et al., 1996] Chawathe, S. S., Rajaraman, A., Garcia-Molina, H., and Widom, J. (1996). Change detection in hierarchically structured information. In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 493–504, Montreal, Quebec, Canada. ACM Press.

[Garofalakis and Kumar, 2003] Garofalakis, M. and Kumar, A. (2003). Correlating XML data streams using tree-edit distance embeddings. In *Proceedings of the Twenty-Second ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS 2003)*, pages 143–154, San Diego, California. ACM Press.

[Garofalakis and Kumar, 2005] Garofalakis, M. and Kumar, A. (2005). XML stream processing using tree-edit distance embeddings. *ACM Transactions on Database Systems*, 30(1):279–332.

[Jiang et al., 1995] Jiang, T., Wang, L., and Zhang, K. (1995). Alignment of trees—an alternative to tree edit. *Theoretical Computer Science*, 143(1):137–148.

[Klein, 1998] Klein, P. N. (1998). Computing the edit-distance between unrooted ordered trees. In *Proceedings of the 6th European Symposium on Algorithms*, volume 1461 of *Lecture Notes in Computer Science*, pages 91–102, Venice, Italy. Springer.

[Lee et al., 2004] Lee, K.-H., Choy, Y.-C., and Cho, S.-B. (2004). An efficient algorithm to compute differences between structured documents. *IEEE Transactions on Knowledge and Data Engineering*, 16(8):965–979.

[Navarro, 2001] Navarro, G. (2001). A guided tour to approximate string matching. *ACM Computing Surveys*, 33(1):31–88.

[Selkow, 1977] Selkow, S. M. (1977). The tree-to-tree editing problem. *Information Processing Letters*, 6(6):184–186.

[Tanaka and Tanaka, 1988] Tanaka, E. and Tanaka, K. (1988). The tree-to-tree editing problem. *Int. Journal of Pattern Recognition and Artificial Intelligence (IJPRAI)*, 2(2):221–240.

[Ukkonen, 1992] Ukkonen, E. (1992). Approximate string-matching with q-grams and maximal matches. *Theoretical Computer Science*, 92(1):191–211.

[Valiente, 2001] Valiente, G. (2001). An efficient bottom-up distance between trees. In *Proceedings of the 8th Symposium on String Processing and Information Retrieval*, pages 212–219, Laguna de San Rafael, Chile. IEEE Computer Science Press.

[Yang, 1991] Yang, W. (1991). Identifying syntactic differences between two programs. *Software—Practice & Experience*, 21(7):739–755.

[Zhang and Shasha, 1989] Zhang, K. and Shasha, D. (1989). Simple fast algorithms for the editing distance between trees and related problems. *SIAM Journal on Computing*, 18(6):1245–1262.