The Clash of Quantum Physics with Gravity

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Abstract. *The author's nonrelativistic model* On the Gravitational Coupling Constant of Elementary Particles *[7] is revisited, especially with regard to the broken scaling-invariance and the breakdown of the superposition principle in the ensuing nonlinear Schrödinger equation. In the light of Weinberg's paper on* Testing Quantum Mechanics *[9], standard homogeneity and linearity of quantum mechanics are secured with sufficient precision in the presence of external fields. The meaning of the so-called Planck-scale is discussed in various contexts. Some computational work on this subject is briefly addressed.*

1. Introduction

1. *Quantum Theory* is based on complex numbers in a linear vector space of at most infinite dimensions.

2. *The Theory of Gravitation*, formulated within the framework of General Relativity, is based on real numbers in a nonlinear 3+1-dimensional NonEuclidean space-time.

Ad 1) Complex numbers are necessary for the pobabilistic interpretation of that theory. In conjunction with linearity one then understands interference of superposed quantum states.

Ad 2) Spacial distances and time-intervals are expressed in terms of real numbers. Since relativistic gravity acts back on itself (equivalence of mass and energy), the correct theory is nonlinear: Starting off with Special Relativity, measuring rods and clocks become distorted to the extent that space-time becomes effectively NonEuclidean [1].

Ad 1) and Ad 2) In quantum theory the fundamental linear Schrödinger equation has a nonrelativistic limit. There is also a nonrelativistic limit in the theory of gravitation, called the Newtonian approximation which happens to be also linear. However, in what follows in this essay, there is no way to generate a unified linear scheme!

One reason for unifying quantum physics with gravity is to understand the interrelationship between fundamental constants: for example, there are

$$\begin{aligned} \hbar &= \text{Planck's constant (divided by } 2\pi) \\ G_0 &= \text{Newton's constant of gravitation} \\ c &= \text{speed of light} \end{aligned}$$

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From these constants we derive a fundamental mass-unit (called the Planck-mass):

$$\sqrt{\frac{\hbar c}{G_0}} \approx 10^{-5}\text{gram} ,$$

which weighs somewhat less than a grownup flea [2].

Penrose has put forward a proposal as to the meaning of the Planck-mass, with G_0 fixed: This fundamental mass might come into play when superpositions of quantum states undergo a *gravitationally induced collapse* towards one single state [2], [3].

In recent years string-theory has come up with intriguing ideas about the the Planck-mass: In models with large (more than 3+1) extra spacial dimensions it is believed that the corresponding Planckian energy-scale ($\approx 10^{16}$ TeV) can effectively be lowered to the TeV-regime, which is now in close reach for accelerators in collision experiments: The actual reason for this is, so the claim, that gravity then freely propagates outside so-called three-branes which are produced in these experiments, see [4] and the references therein. In this case standard General Relativity is supposedly reduced to the low energy-limit of a high-dimensional quantum gravity theory.

I have a different conjecture to this monstrous mass-unit (in reference to the actual mass-scale of elementary particles): The *gravitational constant* is not fundamental, it varies with the epoch [5], [6]. So, the Planck-mass has no universal meaning!

By comparison, my own ideas are rather simple and straightforward, already showing the clash of ordinary linear quantum physics with gravity in the nonrelativistic sector, first developed in [7]:

Let ε be a stationary energy state of an isolated quantum object, localized within a three-dimensional region a, then one naively expects

$$\varepsilon \approx \frac{\hbar^2}{2Ma^2} , \quad \frac{\partial \varepsilon}{\partial a} = 0 ,$$

where M is the *inertial mass*; however, there is no such stationary value, unless $a \to \infty$.

Help comes from *universal gravity*: Since the mass M is also gravitational, there will be a self-energy contribution within that region, s.t.

$$\varepsilon \approx \frac{\hbar^2}{2Ma^2} - \frac{M^2 G_0}{a} , \quad \frac{\partial \varepsilon}{\partial a} = 0 .$$

From this we see at once that for stable stationary states, $\varepsilon < 0$:

$$\frac{M^3 G_0}{\hbar^2} \approx \frac{1}{a} .$$

For isolated elementary particle masses the region of localization is pretty large, so cosmology comes into play: For example, if one puts for $\max(a) \approx 10^{28}$ cm (roughly the Hubble-radius at the present epoch), a tenth of the proton/neutron-mass, possibly a *critical fundamental mass*, emerges, see [5], [8]; at any rate, this mass is of a desirable order of magnitude for elementary particles, and G_0 would thus depend on the epoch!

I am now convinced, contrary to what was said in the original paper [7], that low mass-particles, electrons or neutrinos, etc., do not fit into this nonrelativistic scheme. Incidentally, if the region of localization a for free objects were also bounded from below, s.t. possibly $\min(a) \approx \hbar/Mc$, then, in nonrelativistic approximation, the Planck-mass would be an upper bound for elementary particles at the present epoch!

2. A Nonlinear Quantum Action

In nonrelativistic classical mechanics, a self-interacting system with gravitational mass $\int_\Omega \rho(x)\, d^3x$ in a 3-dimensional region Ω has the self-energy proportional to

$$-\frac{1}{2}\int_\Omega \int_\Omega \rho(x)\rho(y)K(x-y)\, d^3x\, d^3y \; ,$$

where $K(x-y)$ is a positive symmetric twopoint-interaction kernel, and ρ is the mass density. For example, in Einstein-Newtonian gravity:

$$K(x) = \frac{1}{4\pi|x|} \; ,$$

i.e. strictly Coulomb.

Within an extended quantum theory, which should encompass gravity in nonrelativistic approximation, we then postulate a *nonlinear functional* over a complex vector field $|\phi\rangle$ with a Lagrange-parameter e:

$$I_\phi = \left\langle \phi \right| \left(T - \frac{1}{2}\langle\phi|K|\phi\rangle \right) \left| \phi \right\rangle - e\left(\langle\phi|\phi\rangle - \lambda^2\right) \; ,$$

where T denotes the kinetic energy. By minimizing we thus get:

$$\begin{aligned} \frac{\partial I_\phi}{\partial \langle\phi|} &= (H_\phi - e)\; |\phi\rangle = 0 \; , \\ \langle\phi|\phi\rangle &= \lambda^2 \; , \end{aligned}$$

where the Hamiltonian, depending on the state ϕ, is given by $H_\phi = T - \langle\phi|K|\phi\rangle$, with λ^2 denoting the gravitational coupling constant.

Note: Strictly speaking, if λ^2 were zero (no gravity, see remark 2), then $\forall\phi : |\phi\rangle \equiv 0$!

Remark 1: Note that, (compare with [8], [9])

1. this model is not *homogeneous*, i.e. there is no scaling-invariance under the transformation

   $$|\phi\rangle \mapsto \alpha\,|\phi\rangle \; ,$$

   where α is some arbitrary complex number;

2. there is no *linear superposition* of quantum states.

3. A Nonlinear Schrödinger Representation

For the Hamiltonian in the extended Schrödinger picture we should have

$$H_\phi = -\triangle \; - \int_{\mathbb{R}^3} |\phi(y)|^2 K(x-y)\, d^3y \;,$$

where $\triangle$ is the Laplacian, and $\phi(x)$ being the Schrödinger amplitude, s.t. the overall gravitational self-energy in $\mathbb{R}^3$ is given by:

$$-\frac{1}{2}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} |\phi(x)|^2\, |\phi(y)|^2 K(x-y)\, d^3x\, d^3y \;,$$

in correspondence to the above classical expression.

The nonlinear Schrödinger eigenvalue equation then reads [10]:

$$\begin{aligned} [-\triangle - V_\phi(x)]\,\phi(x) &= e\,\phi(x) \;, \\ \int_{\mathbb{R}^3} |\phi(y)|^2 K(x-y)\, d^3y &= V_\phi(x) \;, \\ \int_{\mathbb{R}^3} |\phi(x)|^2\, d^3x &= \lambda^2 \;. \end{aligned}$$

Note that the Lagrange-parameter e is synonymous with the eigenvalue- parameter of the ensuing Schrödinger equation.

Remark 2: After little thought, the above elementary estimate on $\varepsilon < 0$ leads to

$$\lambda^2 = \frac{8\pi G_0 M^3}{\hbar^2} \;.$$

Note that computational work has been done, for example, some time ago by Synge [11], and more recently by Adomian [12], with the simplified assumption that K behaves like a Yukawa-kernel; for general methods of integration, also see [13].

A fundamental theorem: Let the kernel K be a bounded operator, with $e < 0$: then λ^2 is bounded from below; this was proved rigorously for certain finite norms of K in $\mathcal{L}^p(\mathbb{R}^3)$ [10]. There is thus a lower bound on the mass of gravitating particles, in agreement with the original conjecture [7].

Interpretation: If this mass-bound is fundamental, and of the right order of magnitude in the above sense, with the Hubble-radius being the effective range of K: then the *gravitational constant* depends on the cosmological epoch, compare with [5]; thus G_0 drops off as the universe expands!

Note, with G_0 strictly decreasing, the evolution of the universe would probably not be cyclical, which might have some bearing on the second law of thermodynamics [14]. At any rate, the standard Einstein-Friedmann cosmology is at stake!

4. Transition to Ordinary Linear Quantum Physics

One may wonder why ordinary quantum mechanics is extremly applicable, with an unprecedented degree of accuracy [9]:

Since $G_0 \neq 0$, we simply transform $\phi \mapsto \lambda\phi$, then by remark 2 we get the following set of equations, also introducing an *external potential* $U(x)$:

$$\begin{aligned}
\left[-\frac{\hbar^2}{2M}\triangle + U(x) - V_\phi(x)\right]\phi(x) &= \varepsilon\,\phi(x)\ , \\
4\pi M^2 G_0 \int_{\mathbb{R}^3} |\phi(y)|^2 K(x-y)\, d^3y &= V_\phi(x)\ , \\
\int_{\mathbb{R}^3} |\phi(x)|^2\, d^3x &= 1\ .
\end{aligned}$$

Note: For the time dependent scheme, referring to free objects, see the Appendix.

Now, scaling-invariance and linearity can be secured for all practical purposes, since in all tested applications we usually have:

$$|\langle\phi|U|\phi\rangle| \gg |\langle\phi|V_\phi|\phi\rangle|\ .$$

Note that, in this approximation, statistical averaging can be done with sufficient precision in the usual manner, compare with [9].

However, given the hypothesis that gravity changes with the epoch, we cannnot be certain as to the validity of this inequality in the cosmolological past. Here one is reminded of certain critical remarks by the late R.P. Feynman on the law of gravity [15].

Appendix

I present a time-dependent version in the Schrödinger picture, whereby $\phi(x) \mapsto \psi(x,t)$, which also works in the variation of G_0 with the epoch. For free objects we should have:

$$\begin{aligned}
\left(-\frac{\hbar^2}{2M}\triangle - V_\psi\right)\psi &= i\hbar\frac{\partial\psi}{\partial t}\ , \\
4\pi M^2 G_0 \int |\psi(y,t)|^2 K(x-y)\, d^3y &= V_\psi\ , \\
\int |\psi(x,t)|^2\, d^3x &= 1\ .
\end{aligned}$$

The dimensionless formulation:

Let $x \mapsto \mu^{-1}x$, and $t \mapsto \left(\mu^2\hbar/2M\right)^{-1} t$, where μ^{-1} is the effective range of K, bounded in $\mathcal{L}^p(\mathbb{R}^3)$, scaling Coloumb-like: $K \mapsto \mu K$.
With μ varying slowly as ψ evolves unitarily as a function of t, we obtain the following dimensionless scheme:

$$(-\triangle - V_\psi) \;=\; i\frac{\partial\psi}{\partial t}\ ,$$

$$\int |\psi(y,t)|^2 K(x-y)\, d^3y \;=\; V_\psi \;,$$
$$\int |\psi(x,t)|^2\, d^3x \;=\; \frac{\lambda^2}{\mu} \;;$$

thus, according to remark 2, G_0 then varies proportional to μ: Since μ^{-1} is supposedly the Hubble-radius, the *gravitational constant* G_0 decreases in an expanding universe!

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